Innovations

Application of Kamal Transform to Constant Coefficient Ordinary Differential Equations with Piecewise Continuous Functions

[1]Onuoha N.O. & [2]Onyeze V.C.
[1]Department of Mathematics, Imo State University, Owerri, Nigeria
[2]Department of Statistics, Imo State University, Owerri, Nigeria

Abstract: In this paper, Kamal transform was applied to solve constant coefficient ordinary differential equations with piecewise continuous functions. Piecewise continuous functions that are not unit step functions were changed to unit step functions. It has similar characteristics with Laplace transform. The initial conditions were used before the final solution as in Laplace transform, Elzaki transform and Rohit transform; and it reduced the differential equations to algebraic equations which are later solved to obtain the solutions. The results obtained showed that Kamal transform is an efficient and less computational mathematical technique that can be applied to differential equations.

Keywords: Kamal transform, Piecewise continuous functions, unit step functions, ordinary differential equations.

1. Introduction

Piecewise continuous functions are step functions. A piecewise continuous function is a function that is continuous at every point of a given finite closed interval except at finitely many points at which it has jump discontinuity. Differential equations with piecewise continuous functions are applied in physics, electrical engineering, and biology. Abdon and Seda [3] studied piecewise differential equations while Xiao-Ping et al [22] applied piecewise fractional differential equation to COVID-19 infection dynamics.

In solving differential equations with piecewise continuous functions, different analytic techniques have been applied, [21], [5] and [9]. In this research, Kamal transfom is used to solve constant coefficient differential equations with piecewise continuous functions. Kamal transform is a new integral transform introduced by Abdelilah [1]. Kamal transform has been applied to solve partial differential equations [7],[11] and [2]. Mona and Ali [10], studied double Sumudu-Kamal transform with applications. A comparative study of Kamal transform and Laplace transform was done by [8]. Owolabi and Oderinu [13] researched on analytic solution of a generalized nonlinear, Hirota-Satsuma coupled equations using Kamal transform. Sudhanshu [15] applied Kamal transform to Bessel's function. Sudhanshu et al [16], solved linear Volterra integral equations using Kamal transform method. In 2018, [17] and [18] applied Kamal transform in their research. Sudhanshu and Gyanendra [19] found the Kamal transform of error function while Sudhanshu and Swarg [20] applied the new integral transform to Abel's integral equation. Onuoha [12] solved coupled systems of linear ordinary differential equations using Kamal transform technique. Rachana et al [14] researched on Kamal decomposition method and its application to coupled system of nonlinear partial differential equation. Kamal transform has been applied to areas like mechanics [6], cryptography with sandip's method [4] and system of linear Volterra integro-diffferential equation of second kind [23].

2. Kamal Transform of Some Simple Functions

In this section, we find Kamal transform of some simple functions

The Kamal transform of a given function $f(t)$ can be defined as

$$K\{f(t)\} = \int_0^\infty e^{-\frac{1}{v}t} f(t)\,dt = F(v) \tag{1}$$

(i). Let $f(t) = p$, where p is a constant, $p > 0$. Then,

$$K(p) = p\int_0^\infty e^{-\frac{1}{v}t}\,dt = pv \tag{2}$$

(ii). Let $f(t) = t$, then

$$k\{t\} = \int_0^\infty e^{-\frac{1}{v}t}\,t\,dt \tag{3}$$

Integrating equation (3) by parts, we get

$$K\{t\} = v^2 \tag{4}$$

(iii). Let $f(t) = t^2$, then

$$K\{t^2\} = \int_0^\infty e^{-\frac{1}{v}t}\,t^2\,dt \tag{5}$$

Integrating equation (5) by parts, we get

$$K\{t^2\} = 2v^3 \tag{6}$$

In general, if $n>0$ is integer number, then

$$K\left\{t^{n}\right\}=n!v^{n+1}$$

(7)

(iv). Let $f(t)=e^{at}$, then

$$K\left\{e^{at}\right\}=\int_{0}^{\infty}e^{-\frac{1}{v}t}e^{at}dt=\int_{0}^{\infty}e^{-\left(\frac{1}{v}-a\right)t}dt$$

(8)

Integrating equation (8), we get

$$K\left\{e^{at}\right\}=\frac{v}{1-av}$$

(9)

Equation (9) will be useful to find the Kamal transform of:

(v). $K\{\sin at\}=\dfrac{av^{2}}{1+a^{2}v^{2}}$

(10)

(vi). $K\{\cos at\}=\dfrac{v}{1+a^{2}v^{2}}$

(11)

(vii). $K\{\sinh at\}=\dfrac{av^{2}}{1-a^{2}v^{2}}$

(12)

(viii). $K\{\cosh at\}=\dfrac{v}{1-a^{2}v^{2}}$

(13)

3. Kamal Transform of Derivatives

(i). Let the Kamal transform of $y(t)$ be defined as follows:

$$K\left\{y(t)\right\}=\int_{0}^{\infty}e^{-\frac{1}{v}t}ydt=Y(v)$$

(14)

(ii). $K\left\{y'(t)\right\}=\int_{0}^{\infty}e^{-\frac{1}{v}t}y'dt=\dfrac{1}{v}Y(v)-y(0)$

(15)

(iii). $K\left\{y''(t)\right\}=\int_{0}^{\infty}e^{-\frac{1}{v}t}y''dt=\dfrac{1}{v^{2}}Y(v)-\dfrac{1}{v}y(0)-y'(0)$

(16)

(iv). $K\left\{y^{(n)}t\right\}=\int_{0}^{\infty}e^{-\frac{1}{v}t}y^{(n)}dt=\dfrac{1}{v^{n}}Y(n)-\left[\sum_{m=0}^{n-1}v^{m+1-n}y^{(m)}(0)\right]$

(17)

4. Kamal Transform of Piecewise Continuous Function (Unit Step Function)

Let $u(t-c)$ be a unit step function defined as

$$u(t-c)=\begin{cases}0 & t<c\\ 1 & t\geq c\end{cases}$$

(18)

Then,

$$K\{u(t-c)\}=\int_0^\infty e^{-\frac{1}{v}t}u(t-c)\,dt$$

(19)

Following the definition of unit step function, equation (19) becomes

$$K\{u(t-c)\}=\int_c^\infty e^{-\frac{1}{v}t}dt=ve^{-\frac{c}{v}}$$

(20)

Theorem:

Let $ve^{-\frac{c}{v}}$ be the Kamal transform of a unit step function $u(t-c)$, then:

(i). $K\{u(t-c)g(t)\}=e^{-\frac{c}{v}}K\{g(t+c)\}$

(ii). $K\{u(t-c)g(t-c)\}=e^{-\frac{c}{v}}K\{g(t)\}=e^{-\frac{c}{v}}G(v)$

Proof:

(i). Let $g(t)=t$, then $g(t+c)=t+c$

$$K\{u(t-c)t\}=\int_0^\infty e^{-\frac{t}{v}}tu(t-c)\,dt=\int_c^\infty e^{-\frac{t}{v}}tdt=e^{-\frac{c}{v}}\left(cv+v^2\right)$$

$$K\{g(t+c)\}=K\{t+c\}=\int_0^\infty e^{-\frac{t}{v}}tdt+c\int_0^\infty e^{-\frac{t}{v}}dt=v^2+cv$$

Then, $e^{-\frac{c}{v}}K\{t+c\}=e^{-\frac{c}{v}}\left(v^2+cv\right)$

Hence, $K\{u(t-c)g(t)\}=e^{-\frac{c}{v}}K\{g(t+c)\}$

(ii). Let $g(t)=t$, then $g(t-c)=t-c$

$$K\{u(t-c)(t-c)\}=\int_0^\infty e^{-\frac{t}{v}}(t-c)u(t-c)\,dt=\int_c^\infty e^{-\frac{t}{v}}tdt-c\int_c^\infty e^{-\frac{t}{v}}dt=e^{-\frac{c}{v}}\left(cv+v^2\right)-cve^{-\frac{c}{v}}=v^2e^{-\frac{c}{v}}$$

$$K\{g(t)\}=K\{t\}=\int_0^\infty e^{-\frac{t}{v}}tdt=v^2$$

Then, $e^{-\frac{c}{v}}K\{g(t)\}=v^2e^{-\frac{c}{v}}=e^{-\frac{c}{v}}G(v)$

Hence, $K\{u(t-c)g(t-c)\}=e^{-\frac{c}{v}}K\{g(t)\}=e^{-\frac{c}{v}}G(v)$

Next, we apply Kamal transform to differential equations with unit step functions.

Application (1):

Consider the first order differential equation

$$\frac{dy}{dt}+3y=u(t-1)$$

(21a)

$$y(0)=0$$

(21b)

Take Kamal transform of equation (21a)

$$k\left\{\frac{dy}{dt}\right\}+3K\{y\}=K\{u(t-1)\}$$

$$\frac{1}{v}Y(v)-y(0)+3Y(v)=ve^{-\frac{1}{v}}$$

(22)

Applying the initial condition, equation (21a) to equation (22) and simplifying further, we get

$$Y(v)=\frac{v^2e^{-\frac{1}{v}}}{1+3v}$$

(23)

To find the inverse Kamal transform of $Y(v)$, we decompose into partial fraction the right hand side of equation (23) to get

$$Y(v)=\frac{v^2e^{-\frac{1}{v}}}{1+3v}=e^{-\frac{1}{v}}\left\{\frac{v}{3}-\frac{v}{3(1+3v)}\right\}$$

(24)

Then,

$$K^{-1}\{Y(v)\}=\frac{1}{3}K^{-1}\left\{e^{-\frac{1}{v}}\left(v-\frac{v}{1+3v}\right)\right\}$$

(25)

Simplifying equation (25) we get

$$y(t)=\frac{1}{3}u(t-1)-\frac{1}{3}u(t-1)e^{-3(t-1)}$$

(26)

Application (2):

Consider the first order differential equation

$$5\frac{dy}{dt}-y=u(t-4)$$

(27a)

$$y(0)=0$$

(27b)

Take the Kamal transform of equation (27a)

$$5K\left\{\frac{dy}{dt}\right\}-K\{y\}=K\{u(t-4)\}$$

$$5\left\{\frac{1}{v}Y(v)-y(0)\right\}-Y(v)=ve^{-\frac{4}{v}}$$

(28)

Applying the initial condition, equation (27b) to equation (28) and further simplification, we get

$$Y(v)=\frac{v^2e^{-\frac{4}{v}}}{5-v}$$

(29)

Expanding the right hand side of equation (29) into partial fraction, equation (29) becomes

$$Y(v)=e^{-\frac{4}{v}}\left(\frac{v}{1-\frac{1}{5}v}-v\right)$$

(30)

Next, we find the inverse Kamal transform of equation (30)

$$y(t)=u(t-4)\left(e^{\frac{1}{5}(t-4)}-1\right)$$

(31)

Application (3):

Consider the second order differential equation

$$\frac{d^2y}{dt^2}+9y=9u(t-3)$$

(32a)

$$y(0)=\frac{dy(0)}{dt}=0$$

(32b)

Take the Kamal transform of equation (32a)

$$\frac{1}{v^2}Y(v)-\frac{1}{v}y(0)-y'(0)+9Y(v)=9ve^{-\frac{3}{v}}$$

(33)

Applying the initial conditions, equation (32b) to equation (33) and simplifying it, we get

$$Y(v)=\frac{9v^3e^{-\frac{3}{v}}}{1+9v^2}$$

(34)

Expanding the right hand side of equation (34) into partial fractions, we get

$$\frac{9v^3 e^{-\frac{3}{v}}}{1+9v^2} = e^{-\frac{3}{v}}\left(v - \frac{v}{1+9v^2}\right)$$

(35)

Equation (34) can now be written as

$$Y(v) = e^{-\frac{3}{v}}\left(v - \frac{v}{1+9v^2}\right)$$

(36)

Taking the inverse Kamal transform of equation (36), we get

$$y(t) = u(t-3) - u(t-3)\cos 3(t-3)$$

(37)

Application (4):

Consider the second order differential equation

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 3y = g(t)$$

(38a)

$$y(0) = \frac{dy(0)}{dx} = 0$$

(38b)

where $g(t) = \begin{cases} 2 & 0 \le t < 3 \\ -2 & t \ge 3 \end{cases}$

Solving equation (38a) using Kamal transform, we change $g(t)$ to unit step function to get

$$g(t) = 2 - 4u(t-3)$$

(39)

Substituting $g(t)$ into equation (38a), equation (38a) becomes

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 3y = 2 - 4u(t-3)$$

(40a)

$$y(0) = \frac{dy(0)}{dx} = 0$$

(40b)

Take the Kamal transform of equation (40a)

$$\left\{\frac{1}{v^2}Y(v) - \frac{1}{v}y(0) - y'(0)\right\} + 4\left\{\frac{1}{v}Y(v) - y(0)\right\} + 3Y(v) = 2v - 4ve^{-\frac{3}{v}}$$

(41)

Applying the initial conditions, equation (40b) to equation (41) and simplifying, we get

$$Y(v)=\frac{2v^3}{1+4v+3v^2}-\frac{4v^3e^{-\frac{3}{v}}}{1+4v+3v^2}$$

(42)

Before finding the inverse Kamal transform of equation (42), we express the right hand side in partial fractions and write equation (42) in the form

$$Y(v)=\left\{\frac{2v}{3}-\frac{v}{1+v}+\frac{v}{3(1+3v)}\right\}-\left\{\frac{4v}{3}-\frac{2v}{1+v}+\frac{2v}{3(1+3v)}\right\}e^{-\frac{3}{v}}$$

(43)

Then, take the inverse Kamal transform of equation (43) to get

$$y(t)=\frac{1}{3}\left(2-3e^{-t}+e^{-3t}\right)-\frac{2}{3}\left(2-3e^{-t}+e^{-3t}\right)u(t-3)$$

(44)

Application (5):

Consider the second order differential equation

$$\frac{d^2y}{dt^2}=\begin{cases}4-t & 0\le t<5\\ -1 & t\ge 5\end{cases}$$

(45a)

$$y(0)=0,\quad \frac{dy(0)}{dt}=15$$

(45b)

The right hand side of equation (45a) is a piecewise continuous function which when changed to unit step function, equation (45a) becomes

$$\frac{d^2y}{dt^2}=4-t+u(t-5)(t-5)$$

(46)

Taking Kamal transform of equation (46), we get

$$\frac{1}{v^2}Y(v)-\frac{1}{v}y(0)-y'(0)=4v-v^2+2e^{-\frac{5}{v}t}v^3$$

(47)

Applying the initial conditions, equation (45b), we get

$$Y(v)=4v^3-v^4+2e^{-\frac{5}{v}t}v^5+15v^2$$

(48)

Taking inverse Kamal transform of equation (48), we get

$$y(t)=15t+2t^2-\frac{1}{6}t^3+\frac{1}{12}u(t-5)(t-5)^4$$

(49)

Application (6):

Consider the second order differential equation

$$\frac{d^2y}{dt^2}+\frac{dy}{dt}-2y=f(t)$$

(50a)

$$y(0)=\frac{dy(0)}{dt}=0$$

(50b)

where $f(t)$ is a piecewise continuous function defined as

$$f(t)=\begin{cases}4 & 0\le t<4\\ 8-t & 4\le t<8\\ 0 & 8\le t\end{cases}$$

Solving equation (50a) using Kamal transform, we write $f(t)$ as a unit step function.

Writing $f(t)$ as a unit step function, we get

$$f(t)=4-u(t-4)(t-4)+u(t-8)(t-8)$$

Equation (50a) can then be written as

$$\frac{d^2y}{dt^2}+\frac{dy}{dt}-2y=4-u(t-4)(t-4)+u(t-8)(t-8)$$

(51)

Next, we take the Kamal transform of equation (51) and get

$$\left\{\frac{1}{v^2}Y(v)-\frac{1}{v}y(0)-y'(0)\right\}+\left\{\frac{1}{v}Y(v)-y(0)\right\}-2Y(v)=4v-e^{-\frac{4}{v}t}v^2+e^{-\frac{8}{v}t}v^2$$

(52)

Applying the initial conditions, equation (50b) to equation (52) and simplifying, we get

$$Y(v)=\frac{4v^3}{1+v-2v^2}-\frac{e^{-\frac{4}{v}t}v^4}{1+v-2v^2}+\frac{e^{-\frac{8}{v}t}v^4}{1+v-2v^2}$$

(53)

For simplicity, we can write equation (53) in the form

$$Y(v)=F_1(v)-F_2(v)+F_3(v)$$

(54)

where $F_1(v)=\dfrac{4v^3}{1+v-2v^2}$, $F_2(v)=\dfrac{e^{-\frac{4}{v}t}v^4}{1+v-2v^2}$, $F_3(v)=\dfrac{e^{-\frac{8}{v}t}v^4}{1+v-2v^2}$

Finding the inverse Kamal transform of equation (54), we decompose $F_1(v)$, $F_2(v)$ and $F_3(v)$ into partial fractions respectively to get

$$F_1(v) = -2v + \frac{2v}{3(1+2v)} + \frac{4v}{3(1-v)}$$
$$F_2(v) = e^{-\frac{4}{v}t}\left\{-\frac{v}{4} - \frac{v^2}{2} - \frac{v}{12(1+2v)} + \frac{v}{3(1-v)}\right\}$$
$$F_3(v) = e^{-\frac{8}{v}t}\left\{-\frac{v}{4} - \frac{v^2}{2} - \frac{v}{12(1+2v)} + \frac{v}{3(1-v)}\right\}$$

(55)

Equation (54) can then be written as

$$Y(v) = -2v + \frac{2v}{3(1+2v)} + \frac{4v}{3(1-v)} - e^{-\frac{4}{v}t}\left\{-\frac{v}{4} - \frac{v^2}{2} - \frac{v}{12(1+2v)} + \frac{v}{3(1-v)}\right\} +$$
$$e^{-\frac{8}{v}t}\left\{-\frac{v}{4} - \frac{v^2}{2} - \frac{v}{12(1+2v)} + \frac{v}{3(1-v)}\right\}$$

(56)

Then, we find the inverse Kamal transform of equation (56) to get

$$y(t) = -2 + \frac{2}{3}e^{-2t} + \frac{4}{3}e^{t} + u(t-4)\left\{\frac{1}{4} + \frac{1}{2}(t-4) + \frac{1}{12}e^{-2(t-4)} - \frac{1}{3}e^{(t-4)}\right\} -$$
$$u(t-8)\left\{\frac{1}{4} + \frac{1}{2}(t-8) + \frac{1}{12}e^{-2(t-8)} - \frac{1}{3}e^{(t-8)}\right\}$$

(57)

Application (7):

Consider the second order differential equation

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = h(t)$$

(58a)

$$y(0) = 0,\ \frac{dy(0)}{dt} = -2$$

(58b)

wherer $h(t) = \begin{cases} 2 & t < 6 \\ t & 6 \le t < 10 \\ 4 & t \ge 10 \end{cases}$

Take Kamal transform of equation (58a)

$$K\left\{\frac{d^2y}{dt^2}\right\} + 3K\left\{\frac{dy}{dt}\right\} + 2y = K\{h(t)\}$$

(59)

$h(t)$ is a piecewise continuous function. We change $h(t)$ to unit step function before we take the Kamal transform. Changing to unit step function, it becomes

$$h(t) = 2 + (t-2)u(t-6) - (t-4)u(t-10)$$

(60)

Substituting $h(t)$ in equation (59), using equation (60), we get

$$K\left\{\frac{d^2y}{dt^2}\right\}+3K\left\{\frac{dy}{dt}\right\}+2y=K\left\{2+(t-2)u(t-6)-(t-4)u(t-10)\right\}$$

(61)

Taking the Kamal transform, equation (61) becomes

$$\left\{\frac{1}{v^2}Y(v)-\frac{1}{v}y(0)-y'(0)\right\}+3\left\{\frac{1}{v}Y(v)-y(0)\right\}+2Y(v)=2v+4ve^{-\frac{6}{v}}+v^2e^{-\frac{6}{v}}-6ve^{-\frac{10}{v}}-v^2e^{-\frac{10}{v}}$$

(62)

Applying the initial conditions, equation (58b) to equation (62) and simplifying further, we get

$$Y(v)=\frac{v^3\left(2+4e^{-\frac{6}{v}}-6e^{-\frac{10}{v}}\right)}{1+3v+2v^2}+\frac{v^4\left(e^{-\frac{6}{v}}-e^{-\frac{10}{v}}\right)}{1+3v+2v^2}-\frac{2v^2}{1+3v+2v^2}$$

(63)

Equation (63) can be written in the form

$$Y(v)=G_1(v)+G_2(v)-G_3(v)$$

(64)

where $G_1(v)=\dfrac{v^3\left(2+4e^{-\frac{6}{v}}-6e^{-\frac{10}{v}}\right)}{1+3v+2v^2},\ G_2(v)=\dfrac{v^4\left(e^{-\frac{6}{v}}-e^{-\frac{10}{v}}\right)}{1+3v+2v^2},\ G_3(v)=\dfrac{2v^2}{1+3v+2v^2}$

Taking the inverse Kamal transform of equation (64), we first express $G_1(v), G_2(v)$ and $G_3(v)$ in partial fractions

$$G_1(v)=\left(v+\frac{v}{1+2v}-\frac{2v}{1+v}\right)+2e^{-\frac{6}{v}}\left(v+\frac{v}{1+2v}-\frac{2v}{1+v}\right)-3e^{-\frac{10}{v}}\left(v+\frac{v}{1+2v}-\frac{2v}{1+v}\right)$$

$$G_2(v)=e^{-\frac{6}{v}}\left(-\frac{3v}{4}+\frac{v^2}{2}-\frac{v}{4(1+2v)}+\frac{v}{1+v}\right)-e^{-\frac{10}{v}}\left(-\frac{3v}{4}+\frac{v^2}{2}-\frac{v}{4(1+2v)}+\frac{v}{1+v}\right)$$

$$G_3(v)=-\frac{2v}{1+2v}+\frac{2v}{1+v}$$

Equation (64) becomes

$$\begin{aligned}Y(v)=&\left(v+\frac{v}{1+2v}-\frac{2v}{1+v}\right)+2e^{-\frac{6}{v}}\left(v+\frac{v}{1+2v}-\frac{2v}{1+v}\right)-3e^{-\frac{10}{v}}\left(v+\frac{v}{1+2v}-\frac{2v}{1+v}\right)+\\&e^{-\frac{6}{v}}\left(-\frac{3v}{4}+\frac{v^2}{2}-\frac{v}{4(1+2v)}+\frac{v}{1+v}\right)-e^{-\frac{10}{v}}\left(-\frac{3v}{4}+\frac{v^2}{2}-\frac{v}{4(1+2v)}+\frac{v}{1+v}\right)-\\&\left\{-\frac{2v}{1+2v}+\frac{2v}{1+v}\right\}\end{aligned}$$

(65)

Next, we take inverse Kamal transform of equation (4) and get

$$y(t)=1+e^{-2t}-2e^{-t}+2u(t-6)\left(1+e^{-2(t-6)}-2e^{-(t-6)}\right)-3u(t-10)\left(1+e^{-2(t-10)}-2e^{-(t-10)}\right)+$$
$$u(t-6)\left(-\frac{3}{4}+\frac{1}{2}(t-6)-\frac{1}{4}e^{-2(t-6)}+e^{-(t-6)}\right)-u(t-10)\left(-\frac{3}{4}+\frac{1}{2}(t-10)-\frac{1}{4}e^{-2(t-10)}+e^{-(t-10)}\right)-$$
$$2\left(e^{-t}-e^{-2t}\right)$$

(66)

5. Conclusion

In this research work, the new integral transform, Kamal transform, is used to solve constant coefficient differential equations with piecewise continuous functions in particular unit step functions. Kamal transforms of simple functions and the unit step functions were defined. Solving the differential equations, the piecewise continuous functions that are not unit step functions were changed to unit step functions, then the Kamal transforms found. Kamal transform were applied to first and second order differential equations. The results obtained showed that Kamal transform is an efficient mathematical tool that can be applied to higher order non-homogeneous constant coefficient differential equations with or without piecewise continuous functions.

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