Innovations

Let's Talk Math: Challenges and Opportunities in Engaging Students in Mathematical Discourse

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Abstract

The importance of effective communication in mathematics and education is indispensable. Myriads of studies have been conducted on the different aspects of mathematics but few studies have tackled the engagement of students in mathematical discourse. As such, in this study, it aims to explore the challenges and opportunities in engaging students in Mathematical discourse of (10) senior high school teachers. It used qualitative-phenomenological design approach. It was found that teachers view mathematical discourse as a means of interaction of students and teachers like collaboration as well it serves as a means of communication through oral and written communication. Furthermore, this study also revealed that problems such as Process-Content dilemma and Interpersonal and Confidence were encountered by the teachers. However, it was also revealed that teachers also gained opportunities in engaging students in mathematical discourse such as to know the comprehension of students, to provide guide for teaching strategies, and to determine student's confidence

Introduction

Communication is essential in mathematics and education. It is a way of sharing ideas and clarifying understanding. Through communication, ideas become objects of reflection, refinement, discussion, and amendment (Sammons, 2018). Whereas math teachers have traditionally focused on teaching content, more challenging standards are encouraging educators to expand their instruction to promote students' mathematical practice skills, most of which depend heavily on learning to communicate effectively in math.

In Mathematics, as in all subject areas, the true essence of teaching is guiding others to greater understanding. Exemplary Math teachers nurture their students' appreciation of the discipline and lead them to an understanding of math that can be applied in diverse situations. This kind of teaching does more than simply impart facts and procedures that are devoid of context or meaning; it taps into the curiosity of learners and offers them opportunities for mathematical exploration, with teachers and learners working collaboratively to construct knowledge (Stylianides, A. J., & Stylianides, G., 2014). Essential to this learning process is effective communication.

One of the challenges in mathematics teaching is to engage students to communicate mathematically. Discourse in mathematics instruction has received considerable attention; however, prompting mathematical discussions and creating an environment that fosters discourse are challenging tasks for teachers (Güçler, B; Wang, S; Kim, D., 2015). Moreover, students who are not used to talking about mathematics may be uncomfortable with or reluctant to participate in discussions. Discourse in mathematics involves expressing and justifying mathematical thinking and ideas. The primary purposes of facilitating discourse are to help students become aware of others' perspectives and strategies and to clarify and expand students' thinking and approaches (National Council of Teachers of Mathematics, 2000).

Current reforms in mathematics education have placed communication at the heart of the learning process (National Council of Teachers of Mathematics, 2000). As a result, there has been an increased interest in issues and questions related to communication around mathematics (Elliot & Kenney, 2016), in how the kinds of verbal interactions frame what mathematics learning occurs (Kim, D.-J., Ferrini-Mundy, J., &Sfard, A., 2012), and general linguistic conditions that exist in classrooms that affect children's learning through talking particularly in the case of second language learners. Also, research on classroom talk, in general, has suggested that teachers' talk often constrains students and students have few opportunities to contribute to the quantity of talk and the substance of talks. Such findings can be interpreted to mean that teachers' talk is negative or irrelevant in the learning process. While mathematics classrooms have begun to shift away from a teacher-centered instructional model, Sfard, A. (2001) suggested that the teacher is still a critical element in the learning environment because he or she is still an actor in it, and consequently, is a participant in the communication that goes on. This is especially true in classrooms that include second language learners and where the medium of instruction is the students' weaker academic language.

In a classroom environment, the teacher plays a vital role in mathematics learning. Communicating mathematically with the students is one of the strategies in learning. Facilitating mathematical discourse has been consistently identified as a high-leverage instructional strategy (Hill, 2016). Accordingly, Nathan, M. & Knuth, E. (2003) stated that the nature of classroom discourse plays a critical role in promoting the kind of learning and thinking that is valued. Researchers have identified certain principles to characterize desirable forms of classroom discourse (Scardamalia, Bereiter, McLean, Swallow, and Woodruff, 2009). Similarly, Lampert (2010) intended for new knowledge in her mathematics classroom to be "constructed as a joint venture in the class rather than as communication from teacher to student". Nathan, M. & Knuth, E., (2003) also believed that mathematics education must use classroom discourse to develop in students the idea of "doing" mathematics, of conjecturing, scrutinizing, and defending one's ideas, as well as learning about it.

The close connection between student engagement in mathematical talk and their mathematical achievement has been well documented. Staples (2008) draws on a cognitive perspective to describe how student engagement in collaborative discourse supports students to construct rich connected conceptual mathematical understandings. Inherent in this process are the opportunities they have to verbalize their reasoning and analyses and critique the reasoning of others. From a sociocultural perspective, Staples (2008) explains how interacting in collaborative discourse allows students to engage in the core practices of mathematics—that is mathematical practices. Mathematical practices encompass a range of actions proficient problem solvers use; the mathematical skills and knowledge that constitutes expertise in learning and using mathematics. They include, for example, such practices as representing, inquiring, justifying and generalizing reasoning (Boaler, 2003; RAND, 2003).

For too long, mathematics education has focused predominantly on students constructing a body of mathematical knowledge. For students to develop robust reasoning processes; however, students need opportunities not only to construct a broad base of conceptual knowledge; they also require ways to build their understanding of mathematical practices (Hunter, 2013; RAND, 2003; Selling, 2016). A large body of empirical and theoretical research has illustrated that mathematical practices are essential in positively

shaping students' mathematical disposition and competence to do mathematics (Boaler, 2003). Selling (2016) proposes that for students to learn to engage in mathematical practices teachers need to be "responsive to students as they support collective participation in mathematical practices" (p. 513).

In the study of Hunter (2010), its hows the need for teachers to respond 'in the moment' to productive student moves which support their engagement in mathematical practices. In line with the study, the current researchers also see the need for direct attention to be given to the mathematical practices and the inquiry and argumentation discourse which supports its development, on the outcomes for diverse learners who may otherwise be marginalized in the mathematics classroom. Teachers, as the more expert members of the community, take a significant role in constructing inclusive classrooms where respectful exchanges of ideas can occur. For teachers to achieve their students actively engaging in extended conversations which involve mathematical inquiry and challenge they need to negotiate the classroom learning context with the students and co-construct classroom, social and mathematical norms (Hunter, 2010) . The social and mathematical norms refer to a network of obligations and expectations which influence and regulate the interactions in the classroom. Social norms provide members of the learning community with guidelines for acceptable ways to participate and communicate mathematical reasoning; mathematical norms relate specifically to the mathematics and being able to construct mathematically acceptable explanations, representations, justifications and generalizations.

Sfard (2008) developed communicational approach to cognition based on the view that there is a strong relationship between mathematical communication and mathematical thinking. In this regard, she considers learning as moving towards a more sophisticated mathematical discourse through participation (Sfard, 2008). In other words, learning mathematics can be interpreted as a process and change in students' engagement in mathematical discourse (Güçler, 2015). From this point of view, the purpose of teaching mathematics is talking and acting in the ways that mathematically competent people talk and act. Talking and acting in the ways that mathematically competent people talk and act requires learners' effective communication with each other's and with their teachers. In this regard, communication will not be regarded as effective unless all the participants feel confident that all parties involved refer to the same things when using the same words. According to Sfard (2008), effectiveness of communication is dependent on the degree of clarity of discursive focus. That is, the consistence between participants' word use is irreplaceable for effectiveness of communication. Sfard (2001) uses two types of analyses named 'focal analysis' and 'preoccupational analysis' to investigate the effectiveness of communication between participants of mathematical discourse. Focal analysis gives information about students' communication with each other or a teacher's communication with his/her students. The effectiveness of communication is determined by the degree of clarity of discursive focus presented in the communication. Based on this assumption, three components of discursive focus as i) pronounced focus, ii) attended focus and iii) intended focus were distinguished. The pronounced focus is the component of "the word used by an interlocutor to identify the object of her attention" (p.304). The attended focus is the component of "what and how we are attending when speaking" (p. 304). The intended focus is the component of "interlocutor's interpretation of the pronounced and attended foci" (p. 304). More than one pronounced and attended foci are important discursive clues for presence of the intended focus. In some cases, although the participants' pronounced and attended foci are different, their intended foci may be the same.

With the literatures mentioned above, there are limited studies that cater mathematical discourse to senior high school students. Since senior high school is a new program in the basic education curriculum in the Philippines, this study would help teachers asses' students in communication skills in mathematics through planning strategies that will help students learn more in a mathematics classroom with the help of engaging students to mathematical discourse.

Problem Statement

The purpose of this qualitative-phenomenological study was to explore the challenges and opportunities in engaging students to mathematical discourse among senior high school students for mathematics teachers at the University of the Cordilleras Senior High School. The study explored the perspective of teachers through the following questions:

- What are the understanding of teachers on mathematical discourse?
- What are the problems encountered by the teachers in engaging students in mathematical discourse?
- What can teacher gain in engaging students in mathematical discourse?

Methodology

The study used qualitative-phenomenological design approach. Since the study sought to understand the challenges and opportunities that the teachers experience throughout their teaching career, it is best to use phenomenological approach. The fundamental goal of phenomenological research is to arrive at a description of the nature of a particular phenomenon (Creswell, 2013). In other words, phenomenological research is searching for the meaning of a true-life experience from the point of view of a person having experienced it (McMillan, 2004).

The respondents of the study were ten (10) senior high school teachers. There are two criteria used in identifying the respondents which are a.) have been teaching mathematics for at least 2 years and b.) had experience in teaching in senior high school.

The instrument used for data gathering of the study is a form of an interview. The researcher used a semi-structured type of interview. For this study, the researchers used a priori code and create codes from the answers of the respondents and analyzed them through thematic analysis.

Findings and Discussion

Knowledge/Views on Mathematical Discourse

For teacher to develop skills in letting the students get involved to mathematical discourse one should know the nature of it and their understanding towards mathematical discourse is very important for better implementation of such.

Interaction of teachers and students

Discourse in the classroom refers to the language teachers and students use to communicate thoughts through written words or conversation. Responses like "*Mathematical discourse is actually what happens in the classroom wherein me as a teacher teach the students and my students also contribute to the learning process in such a way that they are also sharing their ideas in what I am discussing*." and "*Mathematical discourse involves an active exchange of intellectual thoughts or ideas or solutions or even alternatives in Mathematics. It can be expressed in lectures, peer discussions, research presentations and other avenues*." Supports the idea of mathematical discourse as the interaction of teachers and students

Communication

Mathematical discourse for the teachers is a communication process. Responses like "Mathematical discourse is an approach of discussing mathematical concepts through written or oral communication" and

"Mathematical discourse is the communication done in mathematics class" affirms the belief of the teachers about mathematical discourse as a communication system.

Challenges

Involving students to talk and engage in communication is very challenging especially on the part of the teachers. Figure 1 shows the challenges and/or dilemmas faced by teachers in engaging students to mathematical discourse.

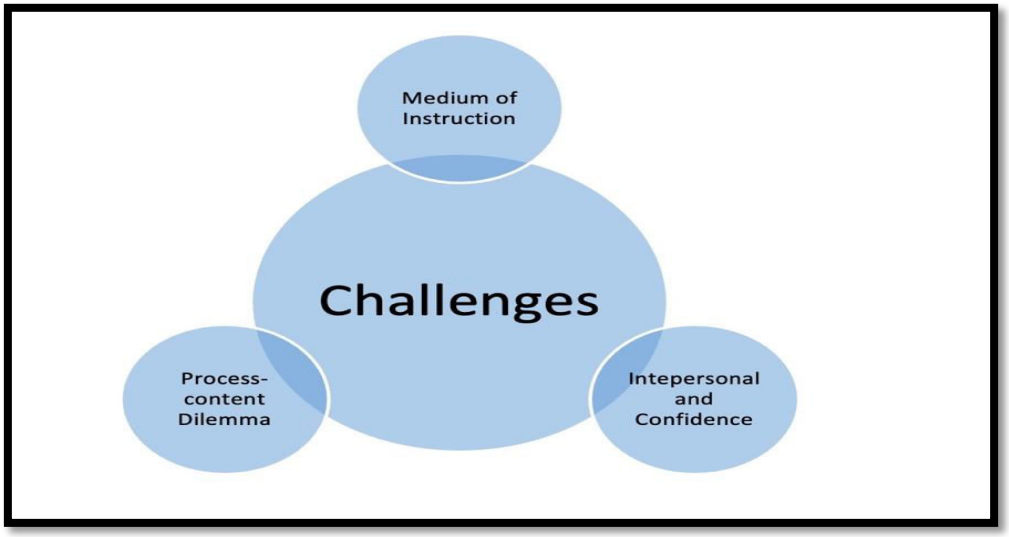

Figure 1: Simulacrum on Challenges in Engaging Students to Mathematical Discourse

Medium of Instruction

The medium of instruction is the language used by the teacher to teach. Teaching the language, or educational content, through the target language increases the amount of exposure the learner gets to it, and the opportunities they have to communicate in it, and therefore to develop their control of it. This one of the challenges that teachers encountered during mathematical discourse This was supported by the participants' answers "Majority of my students during discussion especially if I ask them to explain their work, they tend to ask if they can speak in Filipino the local language of course I gave them the opportunity to at least explain themselves in Filipino every now and then however I always encourage them to at least express themselves in English because that is really the medium of instruction for mathematics." and "…Some of the students are having a difficulty in speaking sometimes if they would recite they could not express themselves in English then sometimes I allow them speak in Filipino."

It is observed that the language in which education is conducted is very important as the selected language may enhance or impede the quality of education. Therefore, language is an important issue, especially in multilingual classrooms where we have students from different linguistic and socio-cultural backgrounds (Md-Ali, R, Mohd-Yusol, F, &Veloo, A, 2014). The language of instruction can also be a problem, especially when the content or concepts being taught are not in the learners' home language. Learning certain subjects, such as Mathematics, in English may be a problem for students whose home language is not English.

While all students can benefit from opportunities to talk about their thinking and reasoning, it may be even more important for students who are not strong in English to engage in discourse that goes beyond superficial language. Dr. Susie Håkansson (2017) suggests that it is important for English language learners to engage in mathematical discourse focused both on receptive language functions (listening and reading) as

well as productive language functions (speaking and writing). Having students who are not proficient in English use more language in the mathematics classroom, rather than less, may be counterintuitive. A teacher's first inclination may be to give such students mathematics involving fewer words, focusing on numbers or symbols. But to do so would be to deny these students the opportunity to engage in rich problems and would put them at a disadvantage, potentially causing them to fall farther behind their English-proficient classmates.

Process-Content dilemma

From the teacher's perspective, a dilemma faces by most of the mathematics teachers' intent upon their dilemma was finding the right balance between creating space for student sharing of mathematical ideas and learning significant mathematics. This shown in the perspective of teachers based on the interview like "Yes, it is challenging, especially to the kind of students we have today. Most of them are passive during class discussions but active when they are with their circle of friends." and "…It's harder for students to be convinced to involve themselves in a discourse. A lot of them would bargain to solve the problem but without any oral explanation something like that. Mathematics became a system of writing and less of oral presentation". While similar to Sfard's (2001) perspective, that mathematical sense making and learning are synonymous, the teacher dilemma described by Sherin is one step removed from Sfard's. Specifically, Sfard's perspective assumes that sharing student strategies and solutions is sense making.

Interpersonal and Confidence

Interpersonal communication and confidence are important for every individual, especially students as individuals who socialize in the school environment. However, these are one of the challenges that teachers encountered during the mathematical discourse.

As the teacher said "During discussions, students are very quiet, but some would freely express the difficulty with the lesson" and "Most of the time, it is challenging to involve students to communicate during my math class because they assume that they will be humiliated if they say the wrong things."

Opportunities

Mathematical discourse provides opportunities for students to know their comprehension level, guide for teaching strategies, and determination of students' confidence. This is also shown in Figure 2.

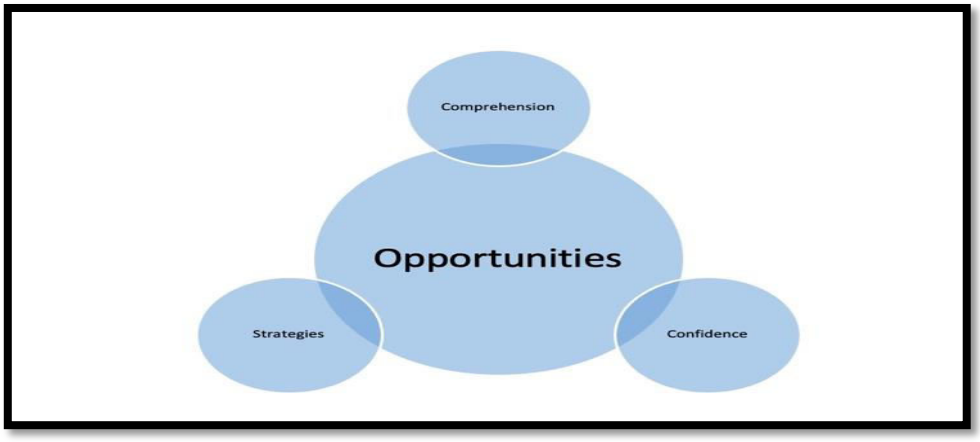

Figure 2: Simulacrum on Opportunities in Engaging Students to Mathematical Discourse

Know the comprehension of students

Mathematical discourse allows teachers to monitor students' dispositions and gauge their developing confidence, interest, and perseverance. Teachers can use this information to determine areas of confusion or frustration in order to decide when an intervention might be needed. They also examine understandings and misconceptions revealed during classroom discussions and adjust lesson plans accordingly. This was apparent in the answer of the participants which states that "I used it when there is a graded recitation which allows me to see how they really or they understood the lesson." And "For mathematical discourse on a positive note, teachers are being clarified on the misconceptions because in my experience I am having my mathematical discourse when I ask for clarifications its not really on the discussion but more on asking clarification for students to respond. So it very important for me to have mathematical discourse during my discussions"

Guide for teaching strategies

Curriculum designed to influence teacher discourse was found to be ineffective (Lloyd, 2008). Conversely, according to Webb, Nemer, and Ing (2006), teacher discourses readily influence student discourses within mathematics classrooms. This was also seen in the answer of one of the participants that "What I do during the first two weeks of the class I try to identify those mathematically inclined and really can express themselves in English or really can engage themselves with mathematics and I tried to have a paired activity and identify those who struggle with mathematics and language or communication so that at least by doing that pair activity most of them would be able to communicate and speak with each other so it could be in vernacular…" and "I used graded recitations however through observing the students behavior during classroom discussion or during discourse this allows me to design a strategy that will involve them in sharing their ideas maybe in problem solving or even during quizzes."

Also, it was identified during the interview that effective strategies like oral presentation, graded recitation and pair and group work were utilized by the teachers in involving students to mathematical discourse.

Determine student's confidence

To engage students in productive mathematics discussions, it is important to establish a learning environment that welcomes student involvement. The first step is setting the expectation that every student will contribute to the discourse community thus can contribute also in identifying the confidence of the students in understanding the lesson. This reflects on the answer of one of the participants which states that "I could say that recitation and oral presentation are effective strategies. This allows me to see the confidence of the students in answering the mathematical problems," "They feel less timid in the class and they do better in rationalizing their answers", and "I have used mathematical discourse very often because I want to understand how confident and how they really understand the lesson or the problems posted to them".

Conclusion

In the study, the researchers were able to identify the different challenges and opportunities teachers experience in involving students to mathematical discourse and also further explore the views or knowledge of the teachers in mathematical discourse. It was also explored the different strategies that teachers do in their mathematical discourse. This study shows that teachers have dilemma in involving students in mathematical discourse such as the use of English as medium of language during recitations, process-content dilemma and confidence of the students. However, with the challenges mentioned, it was also found out that

teachers can benefit during mathematical discourse such as they tend to observe students' confidence during discussion, able to design strategies to promote effective communication during mathematics class and able to identify the comprehensions of the students.

Recommendation

For future researchers, the researchers recommend further concepts of mathematical discourse like art of questioning in mathematical discourse. Further, they could also compare how teachers' years of experiences affect the involvement of mathematical discourse in the classroom. And lastly, the future researchers could interview a larger group of teachers to saturate more data.

References:

1. *Boaler, J. (2003). Exploring the nature of mathematical activity: Using theory, research and 'working hypotheses' to broaden conceptions of mathematics knowing. Educational Studies in Mathematics, 51(1–2), 3–21.*
2. *Creswell, J. W. (2013). Qualitative inquiry & research design: Choosing among five approaches. Los Angeles: SAGE Publications.*
3. *Elliott, P. C., & Kenney, P. (2016).*Communication in mathematics, K-12 and beyond*. Reston, VA: National Council of Teachers of Mathematics.*
4. *Güçler, B; Wang, S; Kim, D., (2015). Conceptualizing mathematics as discourse in different educational settings. International Education Studies; ,8,12;*
5. *Hunter, R. (2010). Changing roles and identities in the construction of a community of mathematical inquiry. Journal of Mathematics Teacher Education, 13(5), 397–409.*
6. *Kim, D.-J., Ferrini-Mundy, J., &Sfard, A. (2012). How does language impact the learning of mathematics? Comparison of English and Korean speaking university students' discourses on infinity. International Journal of Educational Research, 51/52, 86-108.*
7. *Lampert, M. (2010). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. American Educational Research Journal, 27, 29–63.*
8. *Lloyd, G. M. (2008). Teaching Mathematics with a New Curriculum: Changes to Classroom Organization and Interactions. Mathematical Thinking & Learning, 10(2), 163-195.*
9. *McMillan, J. H. (2004). Educational research. Fundamentals for the consumer. Boston, MA: Pearson Education.*
10. *Md-Ali, R, Mohd-Yusol, F, &Veloo, A (2014). Mathematics teachers discourse practices in teaching lesson content using non-native language. The European Journal of Social &Behavioural Sciences. Retreived from www.futureacademy.org.uk*
11. *Nathan, M. & Knuth, E., (2003). A study of whole classroom mathematical discourse and teacher change. Cognition and instruction, 21(2), 175–207. Retrieved from website.education.wisc.edu*
12. *National Council of Teachers of Mathematics. (2000).* Principles and standards for school mathematics*. Retrieved from scholar.google.com*
13. *RAND Mathematics Study Panel. (2003). Mathematical proficiency for all students: Towards a strategic research and development program in mathematics education. Santa Monica, CA: RAND.*
14. *Selling, S. (2016). Making mathematical practices explicit in urban middle and high school mathematics classrooms. Journal for Research in Mathematics Education, 47(5), 505–551.*
15. *Sfard, A. (2001). There is more to discourse than meets the ears: Looking at thinking as communicating to learn more about mathematical learning. Educational Studies in Mathematics, 46(1/3), 13-57.*

16. *Sfard, A. (2008). Thinking as communicating: Human development, the growth of discourses and mathematizing. Cambridge, U.K.: Cambridge University Press.*
17. *Staples, M. (2008). Promoting student collaboration in a detracked, heterogeneous secondary mathematics classroom. Journal of Mathematics Teacher Education, 11(5), 349–371.*
18. *Stylianides, A. J., & Stylianides, G. (2014). Impacting positively on students' mathematical problem solving beliefs: An instructional intervention of short duration. The Journal of Mathematical Behavior, 33, 8–29.*
19. *Scardamalia. M., Bereiter, C., McLean, R. S., Swallow, J., & Woodruff, E. (2009). Computer-supported intentional learning environments. Journal of Educational Computing Research, 5, 51–68*
20. *Webb, N. M., Nemer, K. M., & Ing, M. (2006). Small-Group Reflections: Parallels Between Teacher Discourse and Student Behavior in Peer-Directed Groups. Journal of the Learning Sciences, 15(1), 63-119.*

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