Innovations

Interrelationship between Sharpe Index Model and Portfolio – Markowitz Model in the Context of Correlation Coefficient and Market Variance

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Abstract:

Problem: *There are different types of market that do exist which are Monopsony, Duopsony and Oligopsony. If one buyer exists, then it is called as Monopsony. If two buyers are present, then it is called Duopsony. If more than two buyers exist then it is called as Oligopsony. In the same way there are different markets which are classified based on the number of sellers. If number of seller is one then it is called as Monopoly. If the number of sellers is two, then such situation is called as Duopoly. A situation where the number of sellers is more than two is called as Oligopoly. In the context of Duopoly, there are many companies which are present in the market. When analysis of these companies is done it is generally observed that the theories used are Portfolio Markowitz Model. In this model, details of assets with respect to their β values, σ values and correlation coefficient are collected and the final analyses with respect to proportions are made. These proportions help in dividing the available funds in the market. Similarly there is one more theory called, Sharpe Index Model which deals with stocks when the investment is huge i.e. for more than two shares. If the Sharpe Index Model is considered for two stocks than it gives important results This paper deals with identifying effects on parameters Correlation Coefficient and Market Variance which arises when the Sharpe Index Model and Portfolio Markowitz Model are equated under Duopoly Market where the numbers of stocks are two only.* ***Design/ Methodology/ Approach:*** *Applied Research Design, Convenience Sampling, Theoretical Approach* ***Findings:*** *The value of correlation coefficient is* $r_{12} = \beta_1\beta_2\ \sigma^2_m / \sigma_1\sigma_2$ ***Conclusion:*** *Further application of theories of complex numbers in identifying standard deviation and variance with respect to market can be new scope of research.*

Keywords: *Sharpe Index Model. Portfolio Markowitz Model, Correlation Coefficient*

Introduction: Return and risk decides the direction of portfolio movement. This statement holds well under the purview of Portfolio Markowitz model. If there are two stocks, the risk and return are calculated by using Portfolio Markowitz model. Another theory which is most commonly used is Sharpe Index Model. Sharpe Index Model is used with respect to Market return. Specific coefficients of Market return with respect to Specific stock are calculated. When these two theories are put at equilibrium with respect to two stocks, different formula originates with respect to coefficient of correlation.

Literature Review:

1. Portfolio is an important parameter for any investor. Under the Portfolio, Investor constructs a collection of different stocks. These stocks in turn provide security to the investors. The investors generally construct portfolio so that they can get more return on less risk The utility of portfolio construction lies with the theme that it can be used to know whether an investor is risk bearer, risk tolerant or risk recessive. These kind of portfolio investors help the brokerages to deal with insolvable times where brokerages know who is ready to take risk at crash of the market. [oliinyk, viktor & Kozmenko, Olga.(2019)]

2. Portfolio Construction is an important process. This process consists of different stages. At early stage, the need of investment is taken into consideration. Need and objectives of the investment are made clear. These need and objectives define the purpose and way of investment. Purpose of investment is the key factor in analyzing how and when the returns will come back. If the purpose is for short term, then the strategies would differ from the long term purpose)] Portfolio construction for different people are different. This construction differs from different factors point of view. In general short term investment and long term investments are taken into consideration and the process is initiated. [Krishnamoorthy N, Mahabub Basha. (2002)].

3. Under the process of portfolio construction, after deciding the need and objectives of the investment, different avenues are listed along with their risk and return parameters. Then out of these avenues, some are selected. Selection is based on the choices and situations in which the investor is trying to build the portfolio. Selection is truly based on priority towards the future aspects, present situation and past experience. Based on this selection, allocation of the fund is done [Block Frank E (1969).]

4. After allocation of the funds, the total fund is distributed amongst the avenues for certain period of time. This time duration depends on short goals or long term goals. Generally two things come up in this particular regard one is the tactics and another one is the strategy. In tactics short term investment is taken seriously mainly intraday trade etc. Under Strategy, long term investments are carried out. Retirement plans, availability of funds and cash at ease during old ages etc [Kabir Md. Enamul (2017)].

5. For different proportion of investment, different theories are used. Investor always has many options with respect to selecting the stocks. These kind of selections would bring different outcomes. The choice of what kind of stocks and especially how many stocks to be purchased depends with the investor. If the investor selects more than one stock wherein the investor is selecting only two stocks to invest, Portfolio Markowitz theory is used. [Nukula, Vasishta Bhargava, Prasada Rao, S.S. (2021)].

6. Risk is one of the investment parameter whose presence is inevitable. Any kind of investment, always deals with someway of the risk factor. Sometimes it can be as high as possible or as low as possible. When it comes to Risk, there are two types of risks associated with the market. These are Systematic Risk and Unsystematic Risk. Under the systematic risk is the market risk associated broadly on the factors that run the market. Forces which run the market have a direct influence on the Systematic Risk. [Shukla Ajay, Kukreja Kegha (2014)].

7. Unsystematic Risk are the risks which are unique to the company. These risks are the risks which arise due to the internal factors prevalent within the organization. Company decisions, working culture, administrative styles etc determine Unsystematic Risk. Unsystematic Risk is also called as Unique Risk. [Cox, Larry. A, Griepentrog Gary. L. (1988].

8. Measurement of Risk: There are different techniques to measure the risk factor. In these lines, Mathematical Statistics is used. Under Mathematical Statistics, application of Probability techniques are done in order to analyze the statistical data. [Jokhadze Valeriane, Schmidt Wolfgang M. (2020)]

9. [Virat V. Acharya and others (2017)] Formula for evaluating Systematic Risk: It is also known as on Diversifiable risk wherein the risk calculated with the help of the following formula:

$$\mathbf{E_r = R_f + \beta(R_m - R_f)}$$

where

E_r= Expected return of the security

R_f= Risk free rate

R_m=Return of the market portfolio

β= Systematic Risk

10. There are many noted ratios which add to the understanding of the Portfolio. These ratios help in analyzing the portfolio to such an extent that it helps in knowing the factors which have affected the portfolio. Amongst these ratios, they are Sharpe's Ratio, Treynor's Ratio and Jensen's Alpha [Cogneau, Philipee & Hubner, Georges. (2009)]

11. [Schmid, Friedrich & Schmidt, Rafael (2009)] Sharpe's Ratio: it is a ratio which gives an important update regarding to what extent return on investment has been there on the risk free rate of return with respect to the standard deviation of an investment distribution. The formula used for the analysis of this particular ratio is

Sharpe's Ratio= $\mathbf{R_p\text{-}R_f/\sigma_p}$

where R_p= Expected Portfolio Return

R_f= Risk free rate of return

σ_p= Standard Deviation of Portfolio

12. [Hubner, Georges, The Generalized Treynor Ratio: A Note (2003)]Treynor's Ratio: It is the ratio which gives relevant information on the return on risk free rate of return with respect to the beta coefficient of the portfolio. The formula used for the analysis of this particular ratio is:

Treynor's Ratio = $\mathbf{R_p\text{-}R_f/\beta}$

where R_p= Expected Portfolio Return

R_f= Risk free rate of return

β= Beta of the portfolio

13. [Happy Catherine, Robiyanto Robiyanto (2020)] Jensen's Alpha: It is a parameter which tells to what extent is the performance of the portfolio for a particular risk involved. Jensen's Alpha can be calculated by the following formula:

Jensen's Alpha = $\mathbf{R_p\text{-}[R_f+\beta_p(R_m\text{-}R_f)]}$

where

R_p= Expected Portfolio Return

R_f= Risk Free Rate

β_p= Beta of the portfolio

R_m= Expected Market Return

14. Sharpe Index Model: It is one of the most important models used for the analysis under portfolio investments. In this model variance of the portfolio is calculated with the help of expected market variance, amount of portion of stock to be used in investment, Beta coefficient of the stock and residual variance. [Nagendra, Marishetty (2012)]

15. [Dr. Aloysius Edward J. and Prof. Jagadish K.K (2020)] Formula of Sharpe Index Model: According to Sharpe Index Model:

$\boldsymbol{\sigma_p^2 = (\sum X_i\beta_i)^2\sigma^2_m + (\sum X_i^2 e_i^2)}$ **for i=1 and i=N**

where

σ_p^2 = Variance of Portfolio

σ_m^2 = Expected Market Variance of Index

e_i^2= Residual Variance

X_i= Portion of Stock in the portfolio

β_i= Beta Coefficient of i^{th} Stock

16. Future aspects of Sharpe Index Model: Sharpe Index Model acts as a bridge from traditional approach to modern approach. Utility of Sharpe Index model can be seen in building such portfolios where Residual Variance is almost nil. [Ujwala Chitre, Dr Yogesh Puri (2021)]
17. There are many other Portfolio Evaluating Models have been listed in order to evaluate the Optimal Portfolio for a given set of funds and proportion. [Lekovic, Milijan. (2021)]
18. Capital Asset Pricing Model: It is a model which helps in estimating risk and expected return 's relationship. Basically there are assumptions, which are being formulated in order to build the Capital Asset Pricing Model. One is that the investors are risk averse, with freedom of borrowing and lending at a risk free rate of interest. The condition of market is perfect where entry and exit is applicable. [Rossi, Matteo. (2016)]
19. There are many such parameters which are often associated with Capital Asset Pricing Model such as Expectations of investor with respect to the market, direct or indirect tax that has to be filled, costs incurred during the transactions. To take care of this, a statement is made where apart from assumptions, homogeneous expectations, taxes, transaction costs are nullified. Investors are more interested in maximizing returns in a given period of time. [Rossi, Matteo. (2016).]

20. [Valeed A Ansari (2000)] The formula used under the CAPM model:

$$\mathbf{E(R_i) = R_f + [E(R_m)-R_f/\sigma_m^2]\ \sigma_{im}}$$

where

$E(R_i)$ = Expected return on security i

R_f = Risk Free Return

$E(R_m)$= Expected return on Market Portfolio

σ_m^2 = Variance of return on market portfolio

σ_{im}= Covariance of returns between security i and security m

21. When it comes to Capital Asset Pricing Model, there are two lines which are very important. One is SML and another one is CML. Security Market Line is a graphical line which represents relationship between expected return and covariance of I and M security. [James R. Garven (1988)]
22. [Chinh Duc Pham, Le Tan Phuoc (2020)] One of the striking factors which contribute to the CAPM theory is the utility of β measure. β is measured by using

$$\boldsymbol{\beta = \sigma_{im}/\ \sigma_m^2}$$

The measure of β gives the estimate of slope of the regression line

23. The major drawback of CAPM is that some security can get high returns from the market after adjusting with the returns of the market index. It also adds other that the risk factor is not measured wholly. [. Pankaj Chaudhary (2016)]
24. In order to overcome the drawbacks of CAPM, APT was framed. [Gregory Connor, Robert A. Korajczyk (1986)]
25. [M.Ali Khan, Yeneng Sun (1997)] Arbitrage Pricing Theory is a theory developed by Stephen Ross where number of factors is included with respect to different returns. The formula of Arbitrage Pricing Theory is:

$$\mathbf{R_i= a_0 + a_1bi_1+a_2bi_2+a_3bi_3+\ldots\ldots+a_jbi_j}$$

Where R_i = average Expected return

a_1= sensitivity of return to bi_1

bi_1= beta Coefficient relevant to a factor.

26. The drawback of APT is that it uses different factors together which is difficult to gather data. All the factors which are taken for analysis are many times difficult to get the required data. On such cases analysis becomes difficult. Apart from getting the data, another challenge is calculation. Taking all factors into consideration and then calculating becomes tedious work. Due to this many times this theory is not used. [Gur Huberman (1982)]

27. [Mangram, Myles. (2013)] The formula used for the calculation of Portfolio Standard Deviation under Portfolio Markowitz model is $\boldsymbol{\sigma_p = \sqrt{(X_1^2\sigma_1^2+X_2^2\sigma_2^2+2X_1X_2\sigma_1\sigma_2 r_{12})}}$
where
σ_p= Portfolio Standard Deviation
X_1= Percentage of Portfolio value in the Stock X_1
X_2 = Percentage of Portfolio value in the Stock X_2
σ_1= Standard Deviation of Stock X_1
σ_2 = Standard Deviation of Stock X_2
r_{12}= Correlation Coefficient between X_1 and X_2
28. Future Aspects of Portfolio Markowitz Model: [Khambholiya, Krishna. (2020)]
29. Correlation Coefficient is very important when the Portfolio Markowitz model is used. Correlation Coefficient is the parameter which tells whether there is any relationship between two variables or not. [. Richard Taylor(1990)]
30. [Jie Wu, Na li, Yan Zhao, Julie Wang (2022)]Correlation Coefficient is the parameter which justifies dependent variable and independent variable. X is generally taken as independent variable and Y is taken as dependent variable.
Formula for Correlation Coefficient is:

$$\frac{n\sum XY-\sum X\sum Y}{\sqrt{[n\sum X^2-(\sum X)^2][n\sum Y^2-(\sum Y)^2]}}$$

Another Formula for Correlation Coefficient is $\mathbf{r_{12}}$**= Covariance of** $\boldsymbol{X_{12}/\sigma_1\sigma_2}$
31. Covariance is a parameter which determines the associated variance between two parameters X and Y. Covariance determines the associated difference between two variables which are running under regression relationship with each other. [Garcia Asuero, Agustin & Sayago, Ana & Gonzalez, Gustavo, (2006)]
32. Variance of 1st stock and 2nd stock has to be calculated. Variance will lead to measurement of standard deviation of the both the stocks. Calculating variance of both the stocks help in making the understanding much better as compared to other parameters. [Richard G. Brereton (2018)]
33. Standard Deviation of 1st stock and 2nd stock are very important part of Portfolio Markowitz model. Standard Deviation with respect to all stocks helps in analyzing the variance of the entire portfolio. [Lee, Dong & In, Junyong & lee, Sangseok. (2015)]

Objectives:
1. To Study the Sharpe Index Model
2. To Study the Portfolio Markowitz Model
3. To Study the interrelationship between Sharpe Index Model and Portfolio Markowitz Model
4. To Study Correlation Coefficient Formula by Equating Sharpe Index Model and Portfolio Markowitz Model
5. To study Market Variance under different values of Correlation Coefficient by equating Sharpe Index Model and Portfolio Markowitz Model.

Theoretical Framework:
According to Sharpe Index Model: $\sigma_p{}^2= (\sum X_i\beta_i)^2\sigma^2{}_m + (\sum X_i{}^2e_i{}^2)$ for i=1 and i=N
where
$\sigma_p{}^2$ = Variance of Portfolio
$\sigma_m{}^2$ = Expected Market Variance of Index
$e_i{}^2$= Residual Variance
X_i= Portion of Stock in the portfolio
β_i= Beta Coefficient of i^{th} Stock
Above formula's form gets changed when limit is taken from i=1 to i=2

$\sigma_p^2 = (X_1\beta_1 + X_2\beta_2)^2\sigma^2_m + (X_1^2e_1^2 + X_2^2e_2^2)$ -- (1)

According to Portfolio Markowitz Model:

$\sigma_p = \sqrt{(X_1^2\sigma_1^2+X_2^2\sigma_2^2+2X_1X_2\sigma_1\sigma_2r_{12})}$

where

σ_p= Portfolio Standard Deviation

X_1= Percentage of Portfolio value in the Stock X_1

X_2 = Percentage of Portfolio value in the Stock X_2

σ_1= Standard Deviation of Stock X_1

σ_2 = Standard Deviation of Stock X_2

r_{12}= Correlation Coefficient between X_1 and X_2

$\sigma_p^2 = (X_1^2\sigma_1^2+X_2^2\sigma_2^2+2X_1X_2\sigma_1\sigma_2r_{12})$ --- (2)

r_{12} = Covariance/$\sigma_1\sigma_2$

Covariance = $\sigma_1\sigma_2\ r_{12}$--- (3)

Combining (1) and (2)

$(X_1\beta_1 + X_2\beta_2)^2\sigma^2_m + (X_1^2e_1^2 + X_2^2e_2^2) = (X_1^2\sigma_1^2+X_2^2\sigma_2^2+2X_1X_2\sigma_1\sigma_2r_{12})$

$(X_1^2\beta_1^2+X_2^2\beta_2^2+2X_1X_2\beta_1\beta_2)\ \sigma^2_m + (X_1^2e_1^2 + X_2^2e_2^2) = (X_1^2\sigma_1^2+X_2^2\sigma_2^2+2X_1X_2\sigma_1\sigma_2r_{12})$

$X_1^2\beta_1^2\ \sigma^2_m + X_1^2e_1^2+X_2^2\beta_2^2\ \sigma^2_m+X_2^2e_2^2+2X_1X_2\beta_1\beta_2\ \sigma^2_m= X_1^2\sigma_1^2+X_2^2\sigma_2^2+2X_1X_2\sigma_1\sigma_2r_{12}$

$X_1^2(\beta_1^2\ \sigma^2_m+ e_1^2) + X_2^2(\beta_2^2\ \sigma^2_m+e_2^2) + 2X_1X_2\beta_1\beta_2\ \sigma^2_m= X_1^2\sigma_1^2+X_2^2\sigma_2^2+2X_1X_2\sigma_1\sigma_2r_{12}$

Comparing Left hand Side with Right Hand Side

$\beta_1^2\ \sigma^2_m+ e_1^2 = \sigma_1^2$

$\beta_2^2\ \sigma^2_m+e_2^2 = \sigma_2^2$

$\beta_1\beta_2\ \sigma^2_m= \sigma_1\sigma_2r_{12}$

$\sigma_1= \sqrt{\ }\ \beta_1^2\ \sigma^2_m+ e_1^2$

$\sigma_2= \sqrt{\ }\ \beta_2^2\ \sigma^2_m+e_2^2$

$r_{12} =\ \beta_1\beta_2\ \sigma^2_m/\ \sigma_1\sigma_2$-- (4)

Substituting (3) in (4)

Covariance = $\beta_1\beta_2\ \sigma^2_m$

$\beta_1^2\ \sigma^2_m+ e_1^2 = \sigma_1^2$

$\sigma^2_m= (\sigma_1^2- e_1^2)/\ \beta_1^2$-- (5)

$\beta_2^2\ \sigma^2_m+e_2^2 = \sigma_2^2$

$\sigma^2_m= (\sigma_2^2- e_2^2)/\ \beta_2^2$-- (6)

from (5) and (6)

$(\sigma_1^2 - e_1^2)/\ \beta_1^2 = (\sigma_2^2- e_2^2)/\ \beta_2^2$

$(\sigma_1^2 - e_1^2)\ \beta_2^2/\ (\sigma_2^2- e_2^2)\ \beta_1^2 =1$-- (7)

Table 1: Statistical Parameters

Sl. No	Standard Deviation	Variance of each Stock	Correlation Coefficient	Covariance	Variance of the Index
1	$\sigma_1 = \sqrt{\beta_1^2 \sigma^2_m + e_1^2}$	$\sigma_1^2 = \beta_1^2 \sigma^2_m + e_1^2$	$r_{12} = \beta_1\beta_2 \sigma^2_m / \sigma_1\sigma_2$	$Cov = \beta_1\beta_2 \sigma^2_m$	$\sigma^2_m = (\sigma_1^2 - e_1^2) / \beta_1^2$
2	$\sigma_2 = \sqrt{\beta_2^2 \sigma^2_m + e_2^2}$	$\sigma_2^2 = \beta_2^2 \sigma^2_m + e_2^2$			$\sigma^2_m = (\sigma_2^2 - e_2^2) / \beta_2^2$

Special Cases:

1. When $r_{12} = 1$

$\beta_1\beta_2 \sigma^2_m / \sigma_1\sigma_2 = 1$ -- (8)

$\sigma^2_m = \sigma_1\sigma_2 / \beta_1\beta_2$

$\sigma_m = \sqrt{\sigma_1\sigma_2 / \beta_1\beta_2}$

From (7) and (8)

$\beta_1\beta_2 \sigma^2_m / \sigma_1\sigma_2 = (\sigma_1^2 - e_1^2) \beta_2^2 / (\sigma_2^2 - e_2^2) \beta_1^2$

$\sigma^2_m = \sigma_1 \sigma_2 (\sigma_1^2 - e_1^2) \beta_2 / (\sigma_1^2 - e_1^2) \beta_1^3$

2. When $r_{12} = -1$

$\beta_1\beta_2 \sigma^2_m / \sigma_1\sigma_2 = -1$

$\sigma_m = \sqrt{-\sigma_1\sigma_2 / \beta_1\beta_2}$--(9)

$-\beta_1\beta_2 \sigma^2_m / \sigma_1\sigma_2 = 1$-- (10)

from (7) and (10)

$-\beta_1\beta_2 \sigma^2_m / \sigma_1\sigma_2 = (\sigma_1^2 - e_1^2) \beta_2^2 / (\sigma_2^2 - e_2^2) \beta_1^2$

$\sigma^2_m = -\sigma_1 \sigma_2 (\sigma_1^2 - e_1^2) \beta_2 / (\sigma_1^2 - e_1^2) \beta_1^3$

3. When $r_{12} = 0$

$\beta_1\beta_2 \sigma^2_m / \sigma_1\sigma_2 = 0$

Table 2: Conditions of Correlation Coefficient

Sl.No	Conditions of Correlation Coefficient	Formula for σ^2_m	Formula for σ^2_m	Formula for σ_m
1	$r_{12} = 1$	$\sigma^2_m = \sigma_1 \sigma_2 (\sigma_1^2 - e_1^2) \beta_2 / (\sigma_1^2 - e_1^2) \beta_1^3$	$\sigma^2_m = \sigma_1\sigma_2 / \beta_1\beta_2$	$\sigma_m = \sqrt{\sigma_1\sigma_2 / \beta_1\beta_2}$
2	$r_{12} = -1$	$\sigma^2_m = -\sigma_1 \sigma_2 (\sigma_1^2 - e_1^2) \beta_2 / (\sigma_1^2 - e_1^2) \beta_1^3$	$\sigma^2_m = -\sigma_1\sigma_2 / \beta_1\beta_2$	$\sigma_m = \sqrt{-\sigma_1\sigma_2 / \beta_1\beta_2}$
3	$r_{12} = 0$	$\beta_1\beta_2 \sigma^2_m / \sigma_1\sigma_2 = 0$		

Research Methodology:

Research Design: Applied Research: Firstly the theoretical background is developed with the help of

Type of Data: Secondary Data: The available data has been recorded from the relevant source. The closing price of the company stock has been taken for analysis.

Source: www.nse.com

Sampling Design: Convenience Sampling
Sampling Size: 414;
Sample Size:
Cochran's Formula: $Z^2/4e^2$
Cochran's Formula: $1.96*1.96*/4\ (0.05)^2$
Sample Size= 384.15
Interpretation: Past one month data has been taken for the analysis. The timeline for data collection was from 17th May to 16th June 2023. Totally the data was collected for 23 days. 384 divided by 23 gives the value up to 16.69 rounding till 17. As the paper deals with two sellers in the market, so to bring even numbers for analysis 18 companies where taken up. This makes a total of 414 (23 X18). Total of 414 samples have been taken for collection and analysis.
Data Collection: There are 18 different data sets which have been collected. Based on the data, all the parameters are calculated.

Table 3: Stock Price of Different Companies-I

17th May – 16th June	Asian Paints Ltd (Rs)	Berger Paints Ltd (Rs)	Airtel	Bsnl	Dr Reddy	Apollo	SBI	Yes
17	3092.45	620.1	792.45	191.25	4481.50	4482.15	586.30	15.90
18	3109.05	622.75	799.35	192.20	4423.10	4442.00	574.20	15.70
19	3084.45	625.25	805.75	189.20	4391.95	4447.15	575.15	15.65
22	3084.9	626.2	801.85	188.50	4457.65	4605.30	577.15	15.65
23	3120.6	630.2	799.95	201.00	4460.55	4550.40	581.60	15.55
24	3101.5	634.5	801.40	198.00	4519.25	4536.45	582.70	15.60
25	3123.55	641.6	822.70	193	4503.95	4568.15	581.25	15.50
26	3128.4	647.1	817.95	189.90	4530.95	4609.90	586.00	15.60
29	3137.4	640.95	821.25	192.00	4556.25	4640.45	595.00	15.90
30	3144.4	648.85	818.45	190.15	4518.90	4599.15	592.80	15.90
31	3192.95	650.45	849.90	192.90	4501.05	4621.95	579.85	16.20
1	3240.7	650.5	827.95	193.00	4552.05	4814.25	582.75	16.30
2	3237.25	652.45	836.50	197.50	4610.45	4967.30	587.20	16.40
5	3199.75	650.1	834.50	192.00	4603.55	4950.20	587.20	16.15
6	3213.75	652.3	829.45	192.00	4636.65	4940.10	585.35	16.00
7	3224.8	652.55	844.25	190.40	4673.75	5026.45	589.20	16.15
8	3212.25	650.3	836	208.80	4649.40	4926.65	588.50	16.75

9	3180.55	647.7	833.50	199.10	4666.80	4926.70	578.70	16.60
12	3194.75	648.3	836.35	198.00	4675.65	4924.75	578.65	16.40
13	3262.85	669.15	834.35	194.00	4702.00	4951.05	576.40	16.40
14	3270.2	670.3	827.95	195.50	4699.00	5009.00	576.55	16.20
15	3294.35	676.75	829.70	197.40	4803.45	5218.75	566.50	16.10
16	3308.45	672.6	836.45	197.35	4900.00	5206.05	570.00	16.40

Table 4: Stock Price of Different Companies -II

17th May – 16th June	**Hindustan Copper**	**Hind Zinc**	**IOCL**	**BPCL**	**Tata Motors**	**Suzuki Motors (JPY)**	**Britannia**	**HUL**
17	102.65	312.35	87.55	364.75	515.50	4605	4631.90	2661.75
18	101.90	310.30	86.00	360.70	508.45	4660	4564.35	2627.55
19	101.60	308.35	86.05	360.30	524.95	4682	4499.85	2641.45
22	103.70	308.70	87.15	361.60	522.50	4740	4498.95	2638.10
23	102.90	307.55	89.00	366.60	528.50	4683	4521.65	2628.30
24	102.20	306.55	89.65	363.30	520.20	4627	4519.80	2614.95
25	102.85	304.80	89.15	363.30	514.85	4555	4563.30	2597.25
26	106.90	305.60	90.20	364.30	518.45	4534	4596.20	2652.35
29	108.90	306.70	90.05	361.70	521.45	4575	4575.25	2650.30
30	111.10	306.65	90.30	360.40	518.80	4617	4573.25	2656.55
31	113.25	306.00	90.00	363.50	526.30	4602	4657.05	2667.55
1	111.60	305.65	90.50	364.55	535.20	4557	4635.25	2697.90
2	113.80	306.50	89.80	360.05	535.90	4630	4634.15	2716.70
5	114.35	306.90	89.25	356.80	546.25	4827	4682.60	2695.80
6	113.65	307.05	89.40	356.15	555.75	4895	4705.40	2691.15
7	115.15	305.45	90.85	368.10	567.95	4981	4893.15	2716.20
8	113.45	304.20	90.05	361.10	559.70	4932	4875.05	2679.90

9	112.75	300.55	90.30	359.45	562.30	5044	4878.75	2636.20
12	113.40	298.95	91.85	371.65	564.15	5004	4900.25	2643.45
13	113.35	303.85	91.90	373.60	562.20	5022	4940.10	2675.90
14	115.40	302.15	93.15	373.55	570.30	5116	4941.65	2698.75
15	114.25	303.90	92.90	378.90	569.45	5100	4975.45	2689.70
16	115.10	304.80	92.10	376.00	569.85	5048	5040.00	2714.25

Table 5: Stock Price of Different Companies -III

17th May – 16th June	**Dabur**	**Patanjali**
17/5/2023	523.95	948.75
18/5/2023	523.45	959.8
19/5/2023	519.1	952.15
22/5/2023	522.25	947.55
23/5/2023	525.75	993.95
24/5/2023	526.7	1018.35
25/5/2023	530.15	1033.5
26/5/2023	542.1	1027.1
29/5/2023	548.35	1013.15
30/5/2023	549.9	1021.55
31/5/2023	555.4	1047.5
1/6/2023	556.9	1040.5
2/6/2023	557.2	1034
5/6/2023	552.25	1029.5
6/6/2023	547.5	1031
7/6/2023	556.5	1035.6
8/6/2023	552.1	1025.3

9/6/2023	549.95	1024.05
12/6/2023	553.6	1021.3
13/6/2023	560.45	1044.65
14/6/2023	562.35	1094.6
15/6/2023	569.05	1121.25
16/6/2023	571.6	1140

Data Analysis

Case 1: Asian paints and Berger Paints

$\beta_1 = 0.202298$, $\beta_1^2 = 0.040924$

$\sigma_1 = 69.40584$, $\sigma_1^2 = 4817.17$

$\beta_2 = 3.945048$, $\beta_2^2 = 15.56341$

$\sigma_2 = 15.71684$, $\sigma_2^2 = 247.0192$

Covariance of Asian Paints and Berger Paints = 974.5027

$\beta_1\beta_2\, \sigma^2_m = 974.5027$ from table

$0.202298*3.945084*\sigma^2_m = 974.5027$

$0.79808260303*\sigma^2_m = 974.5027$

$\sigma^2_m = 974.5027/0.79808260303$

$\sigma^2_m = 1221.05493378$

Correlation Coefficient between Asian Paints and Berger Paints Ltd (r_{12}) = 0.933956905

From the generated formula

$\mathbf{r_{12}} = \beta_1\beta_2\, \sigma^2_m / \sigma_1\sigma_2$

$\mathbf{r_{12}} = 0.202298*3.945048*1221.05493378/69.40584*15.71684$

r12= 0.893350

Case 2: **Airtel and BSNL**

$\beta_1 = 0.065846$, $\beta_1^2 = 0.004336$

$\sigma_1 = 16.059$, $\sigma_1^2 = 257.8916$

$\beta_2 = 0.775354$, $\beta_2^2 = 0.601174$

$\sigma_2 = 4.679851$, $\sigma_2^2 = 21.90101$

Covariance of Airtel and Bsnl = 16.98103

$\beta_1\beta_2\, \sigma^2_m = 16.98103$ from table

$0.065846*0.775354*\sigma^2_m = 16.98103$

$0.05105395948*\sigma^2_m = 16.98103$

$\sigma^2_m = 16.98103/0.05105395948$

$\sigma^2_m = 332.609462085$

Correlation Coefficient between Airtel and Bsnl (r_{12}) = 0.236221

From the generated formula

$\mathbf{r_{12}} = \beta_1\beta_2\, \sigma^2_m / \sigma_1\sigma_2$

r_{12} = 0.065846* 0.775354*332.609462085/16.059*4.679851
r_{12}= 0.22595060327

Case 3:**Dr Reddy and Apollo**
β_1 =1.75886 , β_1^2= 3.09359
σ_1= 123.7834, σ_1^2= 15322.34
β_2 =0.463729, β_2^2= 0.215045
σ_2=241.0716, σ_2^2= 58115.5
Covariance of Dr Reddy and Apollo = 26949.85
$\beta_1\beta_2\ \sigma^2_m$ = Covariance from table
1.75886*0.463729* σ^2_m = 26949.85
0.81563438894* σ^2_m = 26949.85
σ^2_m = 26949.85/0.81563438894
σ^2_m = 33041.5813328
Correlation Coefficient between Dr Reddy and Apollo (r_{12}) = 0.944176

From the generated formula
r_{12} = $\beta_1\beta_2\ \sigma^2_m/\ \sigma_1\sigma_2$
r_{12} = 1.75886* 0.463729*33041.5813328/123.7834*241.0716
r12= 26949.85/29840.6622914
r12= 0.90312506260

Case 4: SBI and Yes Bank
β_1 =-0.00157784 , β_1^2= 0.00
σ_1= 7.014593765, σ_1^2= 49.20452569
β_2 = -0.59275, β_2^2= 0.35135
σ_2= 0.361909, σ_2^2= 0.130978
Covariance of SBI and Yes Bank = -0.077637051
$\beta_1\beta_2\ \sigma^2_m$ = -0.077637051 from table
-0.00157784*-0.59275* σ^2_m = -0.077637051
0.00093526466* σ^2_m = -0.077637051
σ^2_m = -0.077637051/0.00093526466
σ^2_m = -83.01078221
Correlation Coefficient between SBI and Yes Bank (r_{12}) = -0.031972165
From the generated formula
r_{12} = $\beta_1\beta_2\ \sigma^2_m/\ \sigma_1\sigma_2$
r_{12} = -0.00157784* -0.59275*-83.01078221/7.014593765*0.361909
r12=- 0.07763705099/2.53864461489
r12=-0.3058208720

Case 5: Hindustan Copper and Hind Zinc
β_1 = -0.32726, β_1^2= 0.1071
σ_1= 5.242451, σ_1^2= 27.48329
β_2 = -1.06104, β_2^2= 1.125808
σ_2= 2.911494, σ_2^2= 8.476798
Covariance of Hindustan Copper and Hind Zinc = -8.99423

$\beta_1\beta_2\ \sigma^2_m$ = -8.99423 from table
-0.32726*-1.06104* σ^2_m = -8.99423
0.3472359504* σ^2_m = -8.99423
σ^2_m = -8.99423/0.3472359504
σ^2_m = -25.9023582945
Correlation Coefficient between Hindustan Copper and Hind Zinc (r_{12}) = -0.61605

From the generated formula
r_{12} = $\beta_1\beta_2\ \sigma^2_m/\ \sigma_1\sigma_2$
r_{12} = -0.32726* -1.06104*-25.9023582945/5.242451*2.911494
r_{12}= -0.58926915637

Case 6: IOCL and BPCL
β_1 = 2.129625, β_1^2=4.535302
σ_1= 1.89682, σ_1^2= 3.597925
β_2 = 0.204134 β_2^2= 0.041671
σ_2= 6.126609, σ_2^2= 37.53534
Covariance of IOCL and BPCL = 7.662231
$\beta_1\beta_2\ \sigma^2_m$ = 7.662231 from table
2.129625*0.204134* σ^2_m = 7.662231
0.43472886975* σ^2_m = 7.662231
σ^2_m = 7.662231/0.43472886975
σ^2_m = 17.6253097807
Correlation Coefficient between IOCL and BPCL (r_{12}) = 0.689309

From the generated formula
r_{12} = $\beta_1\beta_2\ \sigma^2_m/\ \sigma_1\sigma_2$
r_{12} = 2.129625*0.204134*17.6253097807/1.89682*6.126609
r_{12}= 7.662231/11.6210744833
r_{12}= 0.65933929009

Case 7: Tata Motors and Suzuki Motors
β_1 = 8.374941, β_1^2= 70.13964
σ_1= 21.82247, σ_1^2= 476.22
β_2 = 0.095707 β_2^2= 0.00916
σ_2= 204.1378 σ_2^2=41672.24
Covariance of tata Motors and Suzuki Motors = 3988.314
$\beta_1\beta_2\ \sigma^2_m$ = 3988.314 from table
8.374941*0.095707* σ^2_m = 3988.314
0.80154047828* σ^2_m = 3988.314
σ^2_m = 3988.314/0.80154047828
σ^2_m = 4975.81108886
Correlation Coefficient between Tata Motors and Suzuki Motors (r_{12}) = 0.935982

From the generated formula
r_{12} = $\beta_1\beta_2\ \sigma^2_m/\ \sigma_1\sigma_2$

r_{12} = 8.374941*0.095707* 4975.81108886/21.82247*204.1378
r12= 0.89528644224

Case 8: Britannia and HUL
β_1 = 0.105811, β_1^2= 0.011196
σ_1= 176.4954, σ_1^2= 31150.63
β_2 =2.854271, β_2^2= 8.146865
σ_2= 33.98224, σ_2^2= 1154.793
Covariance of Britannia and HUL = 3296.092
$\beta_1\beta_2\,\sigma^2_m$ = 3296.092 from table
0.105811*2.854271* σ^2_m = 3296.092
0.30201326878* σ^2_m = 3296.092
σ^2_m = 3296.092/0.30201326878
σ^2_m = 10913.7324108
Correlation Coefficient between Britannia and HUL (r_{12}) = 0.574538

From the generated formula
r_{12} = $\beta_1\beta_2\,\sigma^2_m/\,\sigma_1\sigma_2$
r_{12} = 0.105811*2.854271* 10913.7324108/176.4954*33.98224
r12= 0.55052793816

Case 9: Dabur and Patanjali
β_1 = , 2.508231 β_1^2= 6.291224
σ_1= 15.96806, σ_1^2= 254.9789
β_2 = 0.274276, β_2^2= 0.075227
σ_2= 48.28833, σ_2^2= 2331.763
Covariance of Dabur and Patanjali = 639.5461
$\beta_1\beta_2\,\sigma^2_m$ = 639.5461 from table
2.508231 *0.274276 * σ^2_m = 639.5461
0.68794756575* σ^2_m = 639.5461
σ^2_m = 639.5461/0.68794756575
σ^2_m = 929.643670303
Correlation Coefficient between AP Ltd and BP Ltd (r_{12}) = 0.867127

From the generated formula
r_{12} = $\beta_1\beta_2\,\sigma^2_m/\,\sigma_1\sigma_2$
r_{12} = 2.508231 *0.274276 * 929.643670303/15.96806*48.28833
r_{12}= 639.5461/771.070950739
r_{12}= 0.8294257482

Table 6: T Test

Sl.No	Correlation Coefficient	Correlation Coefficient Value from Generated Formula
1	0.933956905	0.893350
2	0.236221	0.22595060327
3	0.944176	0.90312506260

4	-0.031972165	-0.3058208720
5	-0.61605	-0.58926915637
6	0.689309	0.65933929009
7	0.935982	0.89528644224
8	0.574538	0.55052793816
9	0.867127	0.82942574827

Hypothesis H_0: There is no significant difference between two groups.

H_1: There is a significant difference between two groups.

Paired T Test: 0.104732183; df= n-1=9-1=8; $T_{\text{Table Value}}$= 2.306

P Test: The Probability of Occurrence of an event can be analyzed using P Test.

P (Z= 0.104732183) = 0.0398 + 0.5 = 0.5398

Mann Whitney U Test: 1. Arrange the data in increasing order.

Table 7:1. Data in increasing order

Sl.No	Data	Increasing Order
1	0.933956905	-0.61605
2	0.236221	-0.589269
3	0.944176	-0.305821
4	-0.03197217	-0.031972
5	-0.61605	0.2259506
6	0.689309	0.236221
7	0.935982	0.5505279
8	0.574538	0.574538
9	0.867127	0.6593393
10	0.89335	0.689309
11	0.225950603	0.8294257
12	0.903125063	0.867127
13	-0.305820872	0.89335
14	-0.589269156	0.8952864
15	0.65933929	0.9031251
16	0.895286442	0.9339569
17	0.550527938	0.935982
18	0.829425748	0.944176

Table 8: 2. Ranking the data

Sl.No	Increasing Order	Ranks
1	-0.61605	1
2	-0.589269	2
3	-0.305821	3
4	-0.031972	4
5	0.2259506	5
6	0.236221	6
7	0.5505279	7
8	0.574538	8
9	0.6593393	9

10	0.689309	10
11	0.8294257	11
12	0.867127	12
13	0.89335	13
14	0.8952864	14
15	0.9031251	15
16	0.9339569	16
17	0.935982	17
18	0.944176	18

Table 9: 3. Classifying according to the group and adding ranks

Sl.No	**Data: Group 1**	**Rank**
1	0.933956905	16
2	0.236221	6
3	0.944176	18
4	-0.03197217	4
5	-0.61605	1
6	0.689309	10
7	0.935982	17
8	0.574538	8
9	0.867127	12
Sum		92
Sl.No	**Data: Group 1**	**Rank**
1	0.89335	13
2	0.225950603	5
3	0.903125063	15
4	-0.305820872	3
5	-0.589269156	2
6	0.65933929	9
7	0.895286442	14
8	0.550527938	7
9	0.829425748	11
Sum		79

4. Comparing with formula

$$U_1 = n_1 n_2 + \frac{n_1(n_2+1)}{2} - R_1$$

$U_1 = 9*9 + 9*10/2 - 92$

$U_1 = 34$

$$U_2 = n_1 n_2 + \frac{n_1(n_2+1)}{2} - R_2$$

$U_2 = 9*9 + 9*10/2 - 79$

$U_2 = 126 - 79$

$U_2 = 47$; $U_{Tab} = 21$

Findings and Suggestions:

1. Both the Sharpe Index model and Portfolio Markowitz model have been studied with respect to two stocks.

2. By equating Sharpe Index model and Portfolio Markowitz Model for two stocks new formulas where generated for Statistical Parameters.

3. The formula which provides the values of correlation coefficient is $\mathbf{r_{12} = \beta_1\beta_2\, \sigma^2_m / \sigma_1\sigma_2}$

4. With respect to Correlation Coefficient, new formula that got generated by equating Sharpe Index Model and Portfolio Markowitz Model was comparatively tested with the known formula of Correlation Coefficient by using statistical tests like T Test and Mann Whitney U Test.

5. T Test briefs that Null Hypothesis is accepted where there is no significant difference between the calculated values of correlation coefficient and Values of Correlation coefficient generated by the formula.

6. The possibility of the Occurrence of the T test is calculated using P test.

7. P test reveals that the possibility of Occurrence of the $r_{12} = \beta_1\beta_2\, \sigma^2_m / \sigma_1\sigma_2$ is 0.5378 which is more than 50%.

8. Mann Whitney U Test gives the result that the calculated values of correlation coefficient and generated values of correlation coefficient are same.

9. The analysis also gives the formula for covariance where **Covariance = $\beta_1\beta_2\, \sigma^2_m$**

10. Market Variance Formula with respect to Stock 1**:** $\sigma^2_m = (\sigma_1^2 - e_1^2) / \beta_1^2$

11. Market Variance Formula with respect to stock 2: $\sigma^2_m = (\sigma_2^2 - e_2^2) / \beta_2^2$

12. Under special case for **r_{12} = 1,** Variance of market is $\sigma^2_m = \sigma_1\sigma_2 / \beta_1\beta_2$ and Standard Deviation is $\sigma_m = \sqrt{}\sigma_1\sigma_2 / \beta_1\beta_2$

13. Under special case for **r_{12} = -1,** Variance of market is $\sigma^2_m = -\sigma_1\sigma_2 / \beta_1\beta_2$ and Standard Deviation is $\sigma_m = \sqrt{}-\sigma_1\sigma_2 / \beta_1\beta_2$

14. When r_{12}=-1, Standard Deviation tends to be $\sigma_m = \sqrt{}-\sigma_1\sigma_2 / \beta_1\beta_2$. This result tells that the Standard Deviation can be square root of a negative number. Such representation implies that the $\sigma_m = \sqrt{}-\sigma_1\sigma_2 / \beta_1\beta_2$ is a complex number which resembles X= i where i = $\sqrt{-1}$.

15. Further application of theories of complex numbers in identifying standard deviation and variance with respect to market can be new scope of research.

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