Innovations

Novel Approach for Nonlinear Time-Fractional Sharma-Tasso-Olever Equation Using Imantransform

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Abstract: *In this article, we demonstrated the study of the time-fractional nonlinear SharmaTasso-Olever (STO) equation with different initial conditions. The novel technique, which is the mixture of the q-homotopy analysis method and the new integral transform known as I man transform called, q-homotopy analysis Iman transform method (q-HAATM) implemented to find the adequate approximated solution of the considered problems. The wave solutions of the STO equation play a vital role in the nonlinear wave model for coastal and harbor designs. The demonstration of the considered scheme is done by carrying out some examples of time-fractional STO equations with different initial approximations. q-HAETM offers us to modulate the range of convergence of the series solution using ℏ, called the auxiliary parameter or convergence control parameter. By performing appropriate numerical simulations, the effectiveness and reliability of the considered technique are validated. The implementation of the new integral transform called the Iman transform along with the reliable analytical technique called the q-homotopy analysis method to examine the time-fractional nonlinear STO equation displays the novelty of the presented work. The obtained findings show that the proposed method is very gratifying and examines the complex nonlinear challenges that arise in science and innovation.*

Keywords: *Sharma-Tasso-Olever equation Liouville-Caputo derivative q-homotopy analysis method Iman transform.*

1. Introduction

Fractional calculus (FC) is an incipient tool in the field of mathematics with strong execution in the diverse areas of science and engineering. FC is defined as the generalization of the classical calculus where we study the integral and differential operators of fractional order, even can be lengthened to a complex set. In the past few decades, many mathematical minds have strengthened this concept and designed various fractional differential and integral operators [1, 2]. The progressive functioning of the demonstration of the classical derivatives is done using the nonlocality of the fractional operators. Fractional operators are undeniably used to define sophisticated memory and a range of objects that may be studied using normal mathematical methods such as classical differentialcalculus. Latterly, fractional operators with nonlocality have been demonstrated and foreseen in the absence of a singular kernel. However, we are still at the initial stage of implementing the concept of FC in various areas of research. Nowadays, FC is a very promising tool due to its larger applications in the dynamics of complex nonlinear phenomena. The idea of fractional calculus has its origin in the correspondence between L'hospital and Leibniz. Additionally, it was shown that FC is much more suitable to handle most complex real-world issues than classical calculus. Fractional calculus's richness in applied research has grown over time. Several studies have now proved its potential to deal with a variety of issues., particularly in the fields of science domains like robotics [3], viscoelasticity [4], image processing [5], biological population models [6], and several more [7- 27]. Compare to the integer-order differential equations, fractional counterparts are much more reserved to get adequate exact solutions for highly nonlinear problems. For this purpose, many numerical and analytical techniques are developed to solve this category of problems. Along with the development of the classical theory in physics, the concept of fractional calculus and its operators has dragged much attention due to its importance in applied physics such as plasma physics, chemical kinematics, fluid mechanics, optical fibres, probability, statistics, etc. Although it has a long history, in recent decades, scientists have been attracted to fractional differential equations (FDE) due to its extensive applications in wide areas of science and engineering upon which few systems which are inherently nonlinear in nature are much studied by physicists, mathematicians, engineers, meteorologists, etc. Nonlinear fractional differential equations (NLFDEs) which describe the change in the variables over time was difficult to solve and unpredictable and are most commonly approximated by linear equations. The basic common approach to solve NLFDEs is either to change the variables so that, the solution for the equation will become simpler like the linear equation or transform the problem that can result in a linear equation. Sometimes, the problem will be converted into one or more ordinary differential equation(s) which may or may not be solvable further. For example, weather forecasting is one of the non-linear behaviour systems in which, some parameters are complete of random behaviour, where simple changes in one part of the system produce complex results throughout the system. Resulting in difficulty with accurate long-term weather forecasts even with current advanced technology. Therefore, the investigation of the exact solutions for NLFDEs plays an important role in the study of a nonlinear system of equations such as Navier–Stokes equations of fluid dynamics, Nonlinear optics, Nonlinear Schrödinger equation, Boltzmann equation, General relativity, Van der Pol oscillator, etc. The inquisition of soliton results of complex nonlinear evolution equations has great significance in the examination of the nonlinear field. These solutions are very informative towards the essential nonlinear science aspects. In this article, we are investigating the nonlinear time-fractional STO equation [28] given as follows:

$$D_t^{\alpha} u(x.t) + 3au_x{}^2 + 3au^2u_x + 3auu_{xx} + au_{xxx} = 0\,.\, t > 0.0 < \alpha \leq 1 \quad (1)$$

where a is the random real constant, u is the dependent variable, t and x are the temporal and spatial variables respectively. The STO equation is similar to the KdV equation which can describe evolutionary physics phenomena and interaction with nonlinear waves, like continuum mechanics, fluid dynamics, solitons and turbulence, aerodynamics, etc. The STO equation incorporates the double nonlinear term and linear dispersive term. The solution of the STO equation has been acquired by numerous methods. The Backlund transformation and Hirota's direct method have been implemented to get the fusion and fission of the solitary wave solutions. It's been revealed that the fission of solutions is obtained for a< 0 and when a> 0 waves depict only the fusion of solutions [29]. The potential symmetries and the generalized symmetries of the STO equation are studied in [30, 31]. Furthermore, to examine the soliton solutions of nonlinear PDEs are analyzed by numerous effective methods so far, like Hirota's method [32], Scattering transformation [33], the First integral method [34, 35], Kudryashov method [36], Extended homoclinic test function method [37, 38], Functional variable method [39], Ansatz method and simplest equation approach [40-42], and others. Various researchers across the globe have given many methods and approaches to solve the nonlinear differential equations among which, Sharma–Tasso– Olver equation which is popularly known as the STO equation has not been much investigated. With this motivation, this work highlights the new generalized novel approach for the nonlinear time-fractional Sharma-Tasso-Olever equation using the Iman transform. To solve linear and nonlinear problems, a semianalytical tool, known as the homotopy analysis method (HAM) is a very efficient scheme recommended and demonstrated by Liao [43-45]. Further, for solving nonlinear problems, the qhomotopy analysis method (q-HAM) as a furnished concept of HAM was introduced by El-Tavil and Hussain [46, 47]. Latterly, the combination of the semianalytical schemes with the Laplace transform is hired to scrutinize nonlinear equations such as Abel integral equation [48], nonlinear fractional shock wave equation [49], nonlinear boundary value problem on the semi-infinite domain [50], two-dimensional Burger's equation [51], class of nonlinear differential equations [52], nonlinear fractional Zakharov-Kuznetsov equation [53], fractional Klein-Gordon-Schrödinger equations [54], fractional coupled Burger's equations [55], and so on. The study of the nonlinear STO equation using various numerical and analytical techniques is covered in a large body of literature. The innovative aspect of the current study is the investigation of the nonlinear timefractional Sharma-Tasso-Olever equation utilizing a powerful analytical tool known as the q-homotopy analysis Iman transform method. The primary goal of this work is to use the new integral transform known as the Imantransform to investigate the fractional behaviour of the problem under consideration. The presented work has not been performed before using the considered algorithm. In the present work, we investigate the reliability and effectiveness of the q-homotopy analysis Iman transform method (q-HAATM) [56] for solving the time-fractional nonlinear STO equation. The considered technique is the amalgamation of the Imantransform (AT) scheme and the q-homotopy analysis method (q-HAM). The Iman transform is the new integral transform obtained by the classical Fourier integral to alleviate the procedure of addressing the solutions for ordinary and partial differential equations. The combination of an Imantransform with the decomposition algorithm is applied to solve the numerous nonlinear partial differential equations , the nonlinear regularized long-wave models are studied with the help of Imantransform, and so on. The benefits of the q-HAATM include not requiring discretization, linearization, perturbations, or any rigid assumptions, significantly reducing the complexity of complex computations, promising a wide convergence region, offering a non-local effect, and not requiring complex polynomials, integrations, or physical parameter calculations. To limit the convergence zone and frequent convergence of the obtained solution to a minimum tolerable region, the studied approach is also natured by auxiliary and homotopy parameters. It produces more digestible outcomes for the identical grid point and series solution sequence. Additionally, the technology under consideration preserves greater accuracy despite requiring less time, making it incredibly efficient and trustworthy. The feasibility and optimism of the considered strategy are demonstrated by its capacity to provide highly precise precision, a large convergence range, and a straightforward solution technique. The rest of the work is organized as follows: Section 2 covers prefaces of the fractional integral in ReimannLiouville sense, IT, and Caputo fractional derivative. The fundamental notion of the investigated methodology is explained in Section 3, and the results for the time-fractional STO equation are discussed in Section 4. Plots are used to explain the responsiveness and pattern of the acquired fractional-order findings. The numerical simulations of the results obtained using q-HAATM are cited in comparison with ADM, HPM, and OHAM. The final section contains comments on the findings obtained.

2. Preliminaries Here we present some basic notions of Fractional operators and the Iman transform:

Definition 1. The fractional Riemann-Liouville integral of a function $(t) \in C_\mu$ $(\mu \geq -1)$, is presented [1] by

$$J^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\vartheta)^{\alpha-1} f(\vartheta) d\vartheta \qquad (2)$$

$$J^0 f(t) = f(t) \qquad (3)$$

Definition 2. The derivative with fractional order α of in the Caputo sense [1] is:

$$D_t^\alpha f(t) = \begin{cases} \dfrac{d^n f(t)}{dt^n}, & \alpha = n \in \mathrm{N} \\ \dfrac{1}{\Gamma(\mathrm{n}-\alpha)} \displaystyle\int_0^t (t-\vartheta)^{n-\alpha-1} f^{(n)}(\vartheta) d\vartheta \;, & \alpha = (n-1, n), n \in \mathrm{N} \end{cases} \qquad (4)$$

Definition 3.

The Imantransform (IT) of a function (t) is demarcated as follows [57]:

$$I\{f(t)\} = \tilde{f}(s) = \frac{1}{S^2} \int_0^\infty e^{-S^2 t} f(t) dt$$

The IT of some basic functions are given below [57]

S.no	$f(t)$	$I\{f(t)\}$
1	1	$\frac{1}{S^4}$
2	t	$\frac{1}{S^6}$
3	e^{at}	$\frac{1}{S^2(S^2-a)}$
4	$\sin(at)$	$\frac{a}{S^2(S^4+a^2)}$
5	$\cos(at)$	$\frac{1}{S^4+a^2}$

Definition 4.
The IT of a derivative in Eq. (4) is presented as [60]

$$I\left\{D_t^{\alpha} f(t)\right\} = s^{2\alpha}\tilde{f}(s) - \sum_{r=0}^{n-1}\left(\frac{1}{S^2}\right)^{2-\alpha+r} f^{(r)}(0), \qquad (n-1<\alpha\leq n). \qquad (5)$$

Where $\tilde{f}(s)$ *denoted the IT of the function* $f(t)$.

3. The basic concept of the q-homotopy analysis Iman transform method (q-HAATM)

Consider the following nonlinear fractional PDE involving linear (N) and nonlinear (R) operators to illustrate the basic principle of the considered method:

$$D_t^{\alpha} u(x,t) + Ru(x,t) + Nu(x,t) = f(x,t), 0<\alpha\leq 1 \qquad (6)$$

where $D_t^{\alpha} u(x,t)$is the Liouville-Caputo fractional derivative of $\mathcal{U}(x,t)$, $f(x,t)$ is the source term. Currently, hiring the IT on Eq. (6) leads to

$$\boldsymbol{s}^{2\alpha}\boldsymbol{I}[u(x,t)] - \sum_{k=0}^{n-1}\left(\frac{\mathbf{1}}{\boldsymbol{s}^2}\right)^{2-\alpha+k}\boldsymbol{u}^{(k)}(\boldsymbol{x},\mathbf{0}) + \boldsymbol{I}[Ru(x,t)] + \boldsymbol{I}[Nu(x,t)] = \boldsymbol{I}[f(x,t)] \quad (7)$$

By reducing Eq. (7), we get

$$\boldsymbol{I}[u(x,t)] - \boldsymbol{s}^{-2\alpha}\sum_{k=0}^{n-1}\left(\frac{\mathbf{1}}{\boldsymbol{s}^2}\right)^{2-\alpha+k}\boldsymbol{u}^{(k)}(\boldsymbol{x},\mathbf{0}) + \boldsymbol{s}^{-2\alpha}\{\boldsymbol{I}[Ru(x,t)] + \boldsymbol{I}[Nu(x,t)] - \boldsymbol{I}[f(x,t)]\} = \mathbf{0} \quad (8)$$

The nonlinear operator N is defined under the homotopy analysis approach as follows

$$N[\varphi(x,t,q)] = \boldsymbol{I}[\varphi(x,t,q)] - \boldsymbol{s}^{-2\alpha}\sum_{k=0}^{n-1}\left(\frac{\mathbf{1}}{\boldsymbol{s}^2}\right)^{\alpha-k-1}\varphi^{(k)}\varphi(x,t,q) + \boldsymbol{s}^{-2\alpha}\{\boldsymbol{I}[R\varphi(x,t,q)] + \boldsymbol{I}[N\varphi(x,t,q)] - \boldsymbol{I}[f(x,t)]\} \quad (9)$$

where A is the Iman transform and $\varphi(x,t,q)$ is a real function of x,t, and q(embedding parameter)$\in[0,\frac{1}{n}](n\geq 1)$.

The homotopy is defined as:

$$(1-nq)I[\varphi(x,t,q) - u_0(x,t)] = hqH(x,t)N[\varphi(x,t,q)], \quad (10)$$

where $u_0(x,t)$is an initial guess of $\mathcal{U}(x,t)$,$\hbar\neq 0$ is an auxiliary parameter. For q=0 and q=$1/n$, respectively we have:

$$\varphi(x,t,0) = u_0(x,t)$$

$$\varphi\left(x,t,\frac{1}{n}\right) = u(x,t) \quad (11)$$

As a result, by changing q from 0 to $\frac{1}{n}$, the solution $\varphi(x,t;q)$ converges from u_0 (x,t) to $\mathcal{U}(x,t)$. The function $(x,t;q)$ can then be enlarged with the utilization of the Taylor theorem across q.

$$\varphi(x,t,q) = u_0(x,t) + \sum_{m=1}^{\infty} u_m(x,t)q^m \tag{12}$$

With

$$u_m(x,t) = \frac{1}{m!}\frac{\partial^m \varphi(x,t,q)}{\partial q^m}|_{q=0} \tag{13}$$

The series (12) joins at q=$\frac{1}{n}$, resulting in the fundamental nonlinear equation, and it's one of the solutions of the type, by selecting the$n$ and $\hbar$ (auxiliary parameter) the initial guess u_0 $(x,)$ and $H(x,t)$ properly.

$$u(x,t) = u_0(x,t) + \sum_{m=1}^{\infty} u_m(x,t)\left(\frac{1}{n}\right)^m \tag{14}$$

Then divide by $m!$ by differentiating Eq. (10) m times with respect to q. Finally, we derive the deformation equation of order m as follows for q=0.

$$A[u_m(x,t) - K_m u_{m-1}(x,t)]= hH(x,t)\mathcal{R}_m(\overrightarrow{u_{m-1}}) \tag{15}$$

and the vectors considered in the form as

$$\overrightarrow{u_m} = \{u_0(x,t), u_1(x,t), u_2(x,t), \ldots\ldots, u_m(x,t)\} \tag{16}$$

Eq. (15) is the recursive equation that may be represented by the effect of the inverse Iman transform

$$[u_m(x,t)] = K_m u_{m-1}(x,t)]+ hI^{-1}[H(x,t)\mathcal{R}_m\left(\overrightarrow{u_{m-1}}\right)] \tag{17}$$

Where

$$[\mathcal{R}_m\left(\overrightarrow{u_{m-1}}\right)] = \frac{1}{(m-1)!}\frac{\partial^{m-1} N[\varphi(x,t,q)]}{\partial q^{m-1}}|_{q=0} \tag{18}$$

And

$$K_m = \begin{cases} 0, m \leq 1 \\ n, m > \end{cases} \tag{19}$$

Finally, we find the component-wise q-HAETM series solution using Eq. (17).

4. Solution for nonlinear Sharma-Tasso-Olever equation of fractional order

The investigation of the following examples witnesses the efficacy and resolution of the contemplated scheme.

4.1. Example 1

The Sharma-Tasso-Olever equation

$$D_t^{\alpha} u(x,t) + 3a{u_x}^2 + 3au^2u_x + 3auu_{xx} + au_{xxx} = 0 \qquad (20)$$

with the starting solution

$$u(x,0) = \frac{2k(\tanh(kx) + w)}{w\tanh(kx) + 1} \quad (21)$$

IntroduceIT on Eq. (20) along with the starting solution in (21), which leads to

$$I[u(x,t)] - \frac{1}{s^2}\left\{\frac{2k(\tanh(kx) + w)}{w\tanh(kx) + 1}\right\} + s^{-\alpha} I\{3a{u_x}^2 + 3au^2u_x + 3auu_{xx} + au_{xxx}\} = 0 \qquad (22)$$

The nonlinear operator N is defined as

$$N[\varphi(x,t,q)] = \boldsymbol{I}[\varphi(x,t,q)] - \boldsymbol{s}^{-2}\left\{\frac{2k(\tanh(kx) + w)}{w\tanh(kx) + 1}\right\} + s^{-\alpha} I\left\{3a\frac{\partial\varphi^2(x,t,q)}{\partial x} + 3a\varphi^2(x,t,q)\frac{\partial\varphi(x,t,q)}{\partial x} + 3a\varphi(x,t,q)\frac{\partial\varphi^2(x,t,q)}{\partial x^2} + a\frac{\partial\varphi^3(x,t,q)}{\partial x^3}\right\} \qquad (23)$$

The m[th] order deformation equation is

$$A[u_m(x,t) - K_m u_{m-1}(x,t)] = h(x,t)\mathcal{R}_m(\overrightarrow{u_{m-1}}) \qquad (24)$$

Where

$$\mathcal{R}_m(\overrightarrow{u_{m-1}}) = I[u(x,t) - \left(1 - \frac{K_m}{n}\right)\frac{1}{s^2}\left\{\frac{2k(\tanh(kx) + w)}{wtanh(kx) + 1}\right\}$$
$$+ s^{-\alpha} I \left\{ 3a \sum_{i=0}^{m-1} \frac{\partial u_i}{\partial x}\frac{\partial u_{m-i-1}}{\partial x}\right.$$
$$+ 3a \sum_{i=0}^{m-1}\sum_{j=0}^{i} u_i u_{i-j}\frac{\partial u_{m-i-1}}{\partial x}$$
$$\left. + 3a \sum_{i=0}^{m-1} u_i \frac{\partial^2 u_{m-i-1}}{\partial x^2} + a\frac{\partial^3 u_{m-1}}{\partial x^3}\right\} \tag{25}$$

Apply inverse IT on Eq. (24), we obtain

$$[u_m(x,t)] = K_m u_{m-1}(x,t)] + hI^{-1}[\mathcal{R}_m(\overrightarrow{u_{m-1}})] \tag{26}$$

From Eq. (26), we arrive at:

$$u_0(x,t) = \frac{2k(\tanh(kx) + w)}{w\ tanh(kx) + 1}$$

$$u_1(x,t) = \frac{8ak^4(w^2 - 1)h\ t^{\alpha}}{\Gamma(\alpha + 1)(w\ sinh(kx) + \cosh kx))^2}$$

Finally, after getting further iterative terms, the essential series solution of Eq. (20) is presented by

$$u(x,t) = u_0(x,t) + \sum_{m=1}^{\infty} u_m(x,t)\left(\frac{1}{n}\right)^m \tag{27}$$

By taking n=1, α=1, and $\hbar$=−1 then the attained solution $\sum_{m=1}^{N} u_m(x,t)\left(\frac{1}{n}\right)^m$, will end up with the exact solution

$\boldsymbol{u(x,t)} = \frac{2k\left(\tanh\left(k(x-4ak^2t)\right)+w\right)}{w\tanh\left(k(x-4ak^2t)\right)+1}$ which is of the Sharma-Tasso-Olever equation as $N\to\infty$.

Conclusion

In this paper, we have demonstrated how to solve the nonlinear time-fractional STO equation using the effective q-HAM with the Iman transform. We have examined three examples with distinct starting solutions to prove the significance as well as the effectiveness of the considered scheme. Moreover, we can compare the obtained results with the exact solutions to witness the same. The rate of convergence of the obtained series solution to the exact solution is accelerated with the help of optimal values of convergence control parameter $\hbar$. Presented numerical simulations guarantee results with higher accuracy. The numerical simulations are executed by using the considered technique in comparison with the other schemes like ADM, HPM, and OHAM in terms of approximated errors. The secure outputs indicate that a considered methodology was used to generate a standardized analytical solution. In this study, the detailed analysis of the fractional behaviour of the nonlinear STO equation and its solution is achieved by considering different initial approximations. The process of finding the solution for the considered problem using the Iman transform was effortless. The proposed approach is effective in delivering a simple solution, a critical convergence zone, and a non-local influence. Finally, we claim that our proposed technique is incredibly dependable and can be applied to large study classifications relating to fractional-order nonlinear scientific methods, which aid us in better understanding the nonlinear compound phenomena in linked domains of innovation and science.

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