Innovations

Fuzzy Neutrosophic Supra Dense G_δ-Spaces

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Abstract

In this paper a new class of fuzzy neutrosophic supra topological space, namely fuzzy neutrosophic supra σ-nowhere dense set and fuzzy neutrosophic supra Dense G_δ-spaces, are introduced and studied. Several characterizations of fuzzy neutrosophic supra σ-nowhere dense set and fuzzy neutrosophic supra Dense G_δ-spaces, are established. The condition under which fuzzy neutrosophic supra σ-nowhere dense set become fuzzy neutrosophic supra nowhere dense set, fuzzy neutrosophic supra residual set, fuzzy neutrosophic supra first category and fuzzy neutrosophic supra G_δ-spaces, are obtained.

Keywords: *Fuzzy neutrosophic supra F_σ −set, Fuzzy neutrosophic supra G_δ − set, Fuzzy neutrosophic supra σ-nowhere dense set, Fuzzy neutrosophic supra σ-first and σ-second category, Fuzzy neutrosophic supra σ-residual set, Fuzzy neutrosophic supra Dense G_δ-space.*

1. Introduction

In recent years, fuzzy and neutrosophic set theories have emerged as powerful tools for dealing with uncertainty in various areas of Mathematics and Computer science. Zadeh [17] was first introduced in 1965 the concept of fuzzy sets and fuzzy set operations in his classical paper. The concept of fuzzy topological spaces, introduced by Chang in 1968 [6], has been extensively studied and applied in many fields, including artificial intelligence, decision making and image processing.

Baire spaces, introduced by Haworth and McCoy in 1977 [7], are an important concept in topology. In 1983, Mashhour et al. [8] extended the notion of supra topological spaces. Abd El-Monsef and Ramadan in 1987 [1] investigated the concept of fuzzy supra topological spaces, which has since been further studied by researchers such as Ahmed, Chandra Chetia, and others [2].

Neutrosophic sets have been introduced to the literature by smarandache to handle incomplete, indeterminate, and inconsistent information. The concept of fuzzy supra σ-Baire space are introduced and studied by Poongothai and Thangaraj [9] investigated fuzzy settings on supra σ-Baire spaces.

The purpose of this paper the concept of fuzzy neutrosophic supra σ-nowhere dense set, fuzzy neutrosophic supra σ-first category, fuzzy neutrosophic supra σ-residual and fuzzy neutrosophic supra Dense G_δ-spaces, are introduced and studied. Several characterizations of fuzzy neutrosophic supra σ-nowhere dense set, fuzzy neutrosophic supra σ-first category fuzzy neutrosophic supra σ-residual and fuzzy neutrosophic supra Dense G_δ-spaces are established. The conditions under which fuzzy neutrosophic supra σ-nowhere dense set become fuzzy neutrosophic supra nowhere dense set, fuzzy neutrosophic supra residual, fuzzy neutrosophic supra first category and fuzzy neutrosophic supra Dense G_δ-spaces are obtained. The existence of fuzzy neutrosophic supra nowhere dense set in fuzzy neutrosophic supra Dense G_δ-spaces are established by means of fuzzy neutrosophic supra σ-nowhere dense set. Several examples are given in this concept.

2. Preliminaries

Definition 2.1 [4]:

A fuzzy neutrosophic set A on the universe of discourse X is defined as $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle \; x \in X$ where $T, I, F: X \rightarrow [0,1]$ and $0 \leq \{T_A(x) + I_A(x) + F_A(x)\} \leq 3$.

Definition 2.2 [4]:

A fuzzy neutrosophic set A is a subset of a fuzzy neutrosophic set B (i.e.,) $A \subseteq B$ for all x if $T_A(x) \leq T_B(x), I_A(x) \leq I_B(x), F_A(x) \geq F_B(x)$

Definition 2.3 [4]:

Let X be a non-empty set, and $A = T_A(x), I_A(x), F_A(x)$, $B = T_B(x), I_B(x), F_B(x)$ be two fuzzy neutrosophic sets. Then

$$A \cup B = \langle x, \max\left(T_A(x), T_B(x)\right), \max\left(I_A(x), I_B(x)\right), \min\left(F_A(x), F_B(x)\right)\rangle$$

$$A \cap B = \langle x, \min\left(T_A(x), T_B(x)\right), \min\left(I_A(x), I_B(x)\right), \max\left(F_A(x), F_B(x)\right)\rangle$$

Definition 2.4 [4]:

The difference between two fuzzy neutrosophic sets A and B is defined as $\mathrm{A \backslash B(x)} = \langle \mathrm{x}, \min\left(\mathrm{T_A(x)}, \mathrm{F_B(x)}\right), \min\left(\mathrm{I_A(x)}, 1\text{-}\mathrm{I_B(x)}\right), \max\left(\mathrm{F_A(x)}, \mathrm{T_B(x)}\right)\rangle$

Definition 2.5 [4]:

A fuzzy neutrosophic set A over the universe X is said to be null or empty fuzzy neutrosophic set if $T(x) = 0, I(x) = 0, F_A(x) = 1$ for all $x \in X$ It is denoted by 0_N .

Definition 2.6 [4]:

A fuzzy neutrosophic set A over the universe X is said to be absolute (universe) fuzzy neutrosophic set if $T(x) = 1, I(x) = 1, F_A(x) = 0$ for all $x \in X$ It is denoted by 0_N.

Definition 2.7 [4]:

The complement of a fuzzy neutrosophic set A is denoted by A^C and is defined as

$A^C = \langle T_{A^C}(x), I_{A^C}(x), F_{A^C}(x) \rangle$

where $T_{A^C}(x) = F_A(x), I_{A^C}(x) = 1 - I_A(x), F_{A^C}(x) = T_A(x)$

The complement of fuzzy neutrosophic set A can also be defined as $A^C = 1_N$-A

Definition 2.8 [5]:

A fuzzy neutrosophic topology on a non-empty set X is a τ_N of fuzzy neutrosophic sets in X Satisfying the following axioms.

$(i) 0_N, 1_N \in \tau$

$(ii) A_1 \cap A_2 \in \tau$ $for\ any\ A_1, A_2 \in \tau$

$(iii)\ \cup A_i \in \tau\ for\ any\ arbitrary\ family\ \{A_i : i \in J\} \in \tau$

In this case the pair(X,τ) is called fuzzy neutrosophic topological space and any fuzzy neutrosophic set in τ is known as fuzzy neutrosophic open set in X.

Definition 2.9 [3]:

A fuzzy neutrosophic supra topology (FNST) a non-empty set X is a τ^μ of fuzzy neutrosophic supra subsets in X satisfying the following axioms.

$(a) 0_N, 1_N \in \tau^\mu$

$(b)\ \cup G_i \in \tau^\mu,\ for\ all\ \{G_i : i \in J\} \subseteq \tau^\mu$

In this case the pair(X,τ^μ) is called a fuzzy neutrosophic supra topological space(FNSTS) and any fuzzy neutrosophic supra set in τ^μ is known as fuzzy neutrosophic supra open set(FNSOS) in X. The element of τ^μ are called open fuzzy neutrosophic supra sets.

The complement of FNSOS in the FNSTS (X,τ^μ) is called fuzzy neutrosophic supra closed set (FNSCS).

Definition 2.10 [3]:

Let (X,τ^{μ}) be FNSTS and P=(y,T_P,I_P,F_P) be a FNSS in X then the fuzzy neutrosophic supra interior (FNSI) and fuzzy neutrosophic supra closure (FNSC) of P are defined by FNScl(P)= $\cap \{Q: Q \text{ is a } FNSCS \text{ in } X \text{ and } P \subseteq Q\}$.

FNS int(P) = $\cup \{R: R \text{ is a } FNSOS \text{ in } X \text{ and } R \subseteq P\}$

Now that Cl(P) is a FNSCS & int(P) is a FNSOS in X, Further

1. P is FNSCS in X iff cl(P) =P

2. P is a FNSOS in X iff int(P)=P.

Definition 2.11 [3]:

Let (X,τ^{μ}) be a FNSTS over X. then the following properties hold.

1. FNScl(CO(P))=CO(FNS int(P))

2. FNS int(CO(P))=CO(FNScl(P)).

Definition 2.12 [5]:

The complement A^C of a fuzzy neutrosophic set A in a fuzzy neutrosophic topological space (X,τ) is called fuzzy neutrosophic closed set in X.

Definition 2.13 [13]:

A fuzzy neutrosophic set X_N^* in fuzzy neutrosophic supra topological space (X_N*,T_N*) is called a fuzzy neutrosophic supra nowhere dense set if there exists no non-zero fuzzy supra open set Y_N^* in (X_N*,T_N*) such that Y_N^*<$fn(cl^*l(X_N^*))$=0, in (X_N*,T_N*). That is $fn(\text{int}^* cl^*(X_N^*)) = 0$ in (X_N*,T_N*).

Definition 2.14 [13]:

A fuzzy neutrosophic supra set X_N^* in a FNSTS (X_N*,T_N*) is called as neutrosophic supra first category set if $X_N^* = \vee_{i=1}^{\infty}(X_{N_i}^*)$'s are neutrosophic supra fuzzy nowhere dense set in (X_N*,T_N*). Any other fuzzy set in (X_N*,T_N*) is said to be neutrosophic supra fuzzy second category.

Definition 2.15 [13]:

A fuzzy neutrosophic supra topological space (X_N*,T_N*) is called a fuzzy neutrosophic supra first category if $\vee_{i=1}^{\infty}(X_{N_i}^*)$ =1, where$(X_{N_i}^*)$'s are fuzzy neutrosophic supra nowhere dense set in (X_N*,T_N*). A topological space which is not of fuzzy neutrosophic supra first category, is said to be fuzzy neutrosophic second category.

Definition 2.16 [13]:

Let X_N^* a fuzzy neutrosophic supra first category set in (X_N*,T_N*), Then $1 - X_N^*$ is called a fuzzy neutrosophic supra residual set in (X_N*,T_N*).

Definition 2.17 [13]:

Let (X_N*,T_N*) be a fuzzy neutrosophic supra topological spaces. Then (X_N*,T_N*) is called fuzzy neutrosophic Baire space if $fn(int^*(\vee_{i=1}^{\infty}(X_{N\,i}^*)))$ =0, where $(X_{N\,i}^*)$'s are fuzzy neutrosophic supra nowhere dense sets in (X_N*,T_N*).

Definition 2.18 [14]

Let (X_N*,T_N*) be a fuzzy neutrosophic supra topological space. A fuzzy set $A_N{}^*$ in (X_N*,T_N*) is called a fuzzy neutrosophic supra residual set if $A_N{}^* = \wedge_{i=1}^{\infty} A_{N_i}{}^*$, where the fuzzy sets $(A_{N_i}{}^*)$,s are such that $fn\ cl^*\left[fn\ int^*\left(A_{N_i}{}^*\right)\right] = 1_N$ in (X_N*,T_N*).

3. Fuzzy Neutrosophic Supra σ-Nowhere Dense Set

Definition 3.1

A fuzzy neutrosophic supra set $A_N{}^*$ in a fuzzy neutrosophic supra topological space (X_N*,T_N*) is called a fuzzy neutrosophic supra σ-nowhere dense set if $A_N{}^*$ is a fuzzy neutrosophic supra F_σ-set in (X_N*,T_N*) such that $int(A_N{}^*) = 0$

Example 3.2

Let $X_N{}^* = \{a, b, c\}$. The fuzzy neutrosophic supra sets $A_N{}^*, B_N{}^*$ and $C_N{}^*$ on $X_N{}^*$ as follows:

$$A_N{}^* = \left(X, \left(\frac{a}{0.3}, \frac{b}{0.7}, \frac{c}{0.4}\right), \left(\frac{a}{0.7}, \frac{b}{0.4}, \frac{c}{0.4}\right), \left(\frac{a}{0.4}, \frac{b}{0.7}, \frac{c}{0.3}\right)\right),$$

$$B_N{}^* = \left(X, \left(\frac{a}{0.5}, \frac{b}{0.7}, \frac{c}{0.8}\right), \left(\frac{a}{0.4}, \frac{b}{0.8}, \frac{c}{0.5}\right), \left(\frac{a}{0.8}, \frac{b}{0.5}, \frac{c}{0.5}\right)\right),$$

$$C_N{}^* = \left(X, \left(\frac{a}{0.2}, \frac{b}{0.4}, \frac{c}{0.6}\right), \left(\frac{a}{0.4}, \frac{b}{0.6}, \frac{c}{0.6}\right), \left(\frac{a}{0.2}, \frac{b}{0.6}, \frac{c}{0.6}\right)\right),$$

Then $T^* = \{0, A_N{}^*, B_N{}^*, C_N{}^*, A_N{}^* \vee B_N{}^*, B_N{}^* \vee C_N{}^*, A_N{}^* \vee C_N{}^*, A_N{}^* \wedge B_N{}^*, B_N{}^* \wedge C_N{}^*, A_N{}^* \wedge C_N{}^*, 1\}$. On computation, we see that $E_N{}^* = (1 - B_N{}^*) \vee (1 - A_N{}^* \vee B_N{}^*) \vee (1 - A_N{}^* \vee C_N{}^*)$ this implies that $E_N{}^*$ is a fuzzy neutrosophic supra F_σ-set in (X_N*,T_N*). Then $int^*(E_N{}^*) = 0$ and hence $E_N{}^*$ is a fuzzy neutrosophic supra σ-nowhere dense set in (X_N*,T_N*).

Also, we see that $int^*(F_N{}^*) = (1 - A_N{}^*) \vee (1 - B_N{}^*) \vee (1 - C_N{}^*) = B_N{}^* \wedge C_N{}^* \neq 0$ and hence $int^*(F_N{}^*) \neq 0$ this implies that $F_N{}^*$ is a fuzzy neutrosophic supra F_σ -set in (X_N*,T_N*) but $F_N{}^*$ is not a fuzzy neutrosophic supra σ-nowhere dense set in (X_N*,T_N*).

Proposition 3.3

In a fuzzy neutrosophic supra topological space (X_N*,T_N*), a fuzzy neutrosophic supra set $A_N{}^*$ is a fuzzy neutrosophic supra σ-nowhere dense set in (X_N*,T_N*) if and only if $1 - A_N{}^*$ is a fuzzy neutrosophic supra dense and fuzzy neutrosophic supra G_δ-set in (X_N*,T_N*).

Proof.

Let $A_N{}^*$ be a fuzzy neutrosophic supra σ-nowhere dense set in (X_N*,T_N*). Then $A_N{}^* = \vee_{i=1}^{\infty} A_{N_i}{}^*$, where $1 - A_{N_i}{}^* \in T_N{}^*$, for $i \in I$ and $int^*(A_N{}^*) = 0$. Then $cl^*(1 - A_N{}^*) = 1 - int^*(A_N{}^*) = 1 - 0 = 1$. This implies that $1 - A_N{}^*$ is a fuzzy neutrosophic supra dense set in (X_N*,T_N*). Also $1 - A_N{}^* = 1 - \vee_{i=1}^{\infty}(A_{N_i}{}^*) = \wedge_{i=1}^{\infty} 1 - (A_{N_i}{}^*)$, where $1 - A_{N_i}{}^* \in T_N{}^*$, for $i \in I$. This implies that $1 - A_N{}^*$ is a fuzzy neutrosophic supra G_δ-set in (X_N*,T_N*). Therefore $1 - A_N{}^*$ is a fuzzy neutrosophic supra dense and fuzzy neutrosophic supra G_δ-set in (X_N*,T_N*).

Conversely, let $A_N{}^*$ be a fuzzy neutrosophic supra dense and fuzzy neutrosophic supra G_δ-set in (X_N*,T_N*). Then $A_N{}^* = \wedge_{i=1}^{\infty}(A_{N_i}{}^*)$, where $A_{N_i}{}^* \in T_N{}^*$, for $i \in I$ and $cl^*(A_N{}^*) = 1$., Now $1 - A_N{}^* = 1 - \wedge_{i=1}^{\infty} A_{N_i}{}^* = \vee_{i=1}^{\infty}(1 - A_{N_i}{}^*)$, where $(1 - A_{N_i}{}^*)$'s are fuzzy neutrosophic supra closed set in (X_N*,T_N*). This implies that $1 - A_N{}^*$ is a fuzzy neutrosophic supra F_σ-set in (X_N*,T_N*). Now $int^*(1 - A_N{}^*) = 1 - cl^*(A_N{}^*) = 1 - 1 = 0$. Hence, $1 - A_N{}^*$ is a fuzzy neutrosophic supra F_σ-set in (X_N*,T_N*) and $int^*(1 - A_N{}^*) = 0$. Therefore $1 - A_N{}^*$ is a fuzzy neutrosophic supra σ-nowhere dense set in (X_N*,T_N*).

Proposition 3.4

If $A_N{}^*$ is a fuzzy neutrosophic supra dense in a fuzzy neutrosophic supra topological space (X_N*,T_N*) such that $B_N{}^* \leq (1 - A_N{}^*)$, where $B_N{}^*$ is a fuzzy neutrosophic supra F_σ-set in (X_N*,T_N*), then $B_N{}^*$ is a fuzzy neutrosophic supra σ-nowhere dense set in (X_N*,T_N*).

Proof.

Let $A_N{}^*$ is a fuzzy neutrosophic supra dense in (X_N*,T_N*) such that $B_N{}^* \leq (1 - A_N{}^*)$. Then $cl^*(A_N{}^*) = 1$ and $B_N{}^* \leq (1 - A_N{}^*)$. Now $B_N{}^* \leq (1 - A_N{}^*)$, implies that $int^*(B_N{}^*) \leq int^*(1 - A_N{}^*) = cl^*(A_N{}^*) = 1 - 1 = 0$ and hence $int^*(B_N{}^*) \leq 0$. That is $int^*(B_N{}^*) = 0$. Since $B_N{}^*$ is a fuzzy neutrosophic supra F_σ-set in (X_N*,T_N*) and $int^*(B_N{}^*) = 0$, $B_N{}^*$ is a fuzzy neutrosophic supra σ-nowhere dense set in (X_N*,T_N*).

Proposition 3.5

If $A_N{}^*$ is a fuzzy neutrosophic supra F_σ-set and fuzzy neutrosophic supra nowhere dense in a fuzzy neutrosophic supra topological space (X_N*,T_N*), then $A_N{}^*$ is a fuzzy neutrosophic supra σ-nowhere dense set in (X_N*,T_N*).

Proof.

It is clear that $A_N{}^* \leq cl^*(A_N{}^*)$ for any fuzzy neutrosophic supra set in $(X_N{}^*,T_N{}^*)$. Then $int^*(A_N{}^*) \leq int^*cl^*(A_N{}^*)$. Since $A_N{}^*$ is a fuzzy neutrosophic supra nowhere dense set in $(X_N{}^*,T_N{}^*)$, $int^*cl^*(A_N{}^*) = 0$ and hence $int^*(A_N{}^*) = 0$, Now $A_N{}^*$ is a fuzzy neutrosophic supra F_σ-set in $(X_N{}^*,T_N{}^*)$ and $int^*(A_N{}^*) = 0$, implies that $A_N{}^*$ is a fuzzy neutrosophic supra σ-nowhere dense set in $(X_N{}^*,T_N{}^*)$.

Proposition 3.6

If $A_N{}^* \leq B_N{}^*$, where $A_N{}^*$ is a fuzzy neutrosophic supra F_σ-set and $B_N{}^*$ is a fuzzy neutrosophic supra σ-nowhere dense set in a fuzzy neutrosophic supra topological space $(X_N{}^*,T_N{}^*)$, then $A_N{}^*$ is also a fuzzy neutrosophic supra σ-nowhere dense set in $(X_N{}^*,T_N{}^*)$.

Proof.

Now $A_N{}^* \leq B_N{}^*$ implies that $int^*(A_N{}^*) \leq int^*(B_N{}^*)$. Since $B_N{}^*$ is a fuzzy neutrosophic supra σ-nowhere dense set in $(X_N{}^*,T_N{}^*)$, $B_N{}^*$ is a fuzzy neutrosophic supra F_σ-set in $(X_N{}^*,T_N{}^*)$ such that $int^*(B_N{}^*) = 0$. Then $int^*(A_N{}^*) \leq 0$. That is., $int^*(A_N{}^*) = 0$. Hence $A_N{}^*$ is a fuzzy neutrosophic supra F_σ-set in $(X_N{}^*,T_N{}^*)$ such that $int^*(A_N{}^*) \leq 0$. Therefore $A_N{}^*$ is a fuzzy neutrosophic supra σ-nowhere dense set in $(X_N{}^*,T_N{}^*)$.

Proposition 3.7

If $A_N{}^*$ is a fuzzy neutrosophic supra σ-nowhere dense set in a fuzzy neutrosophic supra topological space $(X_N{}^*,T_N{}^*)$, then $1 - A_N{}^* = \wedge_{i-1}^{\infty}(B_{N_i}{}^*)$, where $cl^*(B_{N_i}{}^*) = 1$.

Proof.

Let $A_N{}^*$ be a fuzzy neutrosophic supra σ-nowhere dense set in $(X_N{}^*,T_N{}^*)$, Then $A_N{}^*$ is a fuzzy neutrosophic F_σ-set in $(X_N{}^*,T_N{}^*)$ such that $int^*(A_N{}^*) = 0$. Hence $A_N{}^* = \vee_{i=1}^{\infty}(A_{N_i}{}^*)$, where $1 - A_{N_i}{}^* \in T_N{}^*$ for $i \in I$ and $int^*(A_N{}^*) = 0$. Now $int^*(A_N{}^*) = int^*(\vee_{i=1}^{\infty}(A_{N_i}{}^*)) = 0$, Then $0 = int^*(\vee_{i=1}^{\infty}(A_{N_i}{}^*))$. But, $\vee_{i=1}^{\infty}\left(int^*(A_{N_i}{}^*)\right) \leq int^*(\vee_{i=1}^{\infty}(A_{N_i}{}^*))$. This implies that $\vee_{i=1}^{\infty}\left(int^*(A_{N_i}{}^*)\right) \leq 0$, that is., $\vee_{i=1}^{\infty}\left(int^*(A_{N_i}{}^*)\right) = 0$ and hence $int^*(A_{N_i}{}^*) = 0$. Then $int^*(A_{N_i}{}^*) = 1 - int^*(A_{N_i}{}^*) = 1 - 0 = 1$. Now $1 - A_N{}^* = 1 - \vee_{i=1}^{\infty}(A_{N_i}{}^*) = \wedge_{i-1}^{\infty}(1 - A_{N_i})$, where $cl^*(1 - A_{N_i}{}^*) = 1$. Let $B_{N_i}{}^* = 1 - A_{N_i}{}^*$. Therefore, $1 - A_N{}^* = \wedge_{i-1}^{\infty}(B_{N_i}{}^*)$, where $cl^*(B_{N_i}{}^*) = 1$.

4. Fuzzy Neutrosophic Supra σ- First Category and Fuzzy Neutrosophic Supra σ- Second Category Set

Definition 4.1:

A fuzzy neutrosophic supra set $A_N{}^*$ in a fuzzy neutrosophic supra topological space $(X_N{}^*,T_N{}^*)$ is called a fuzzy neutrosophic supra σ-first category set if $A_N{}^* = \vee_{i=1}^{\infty}(A_{N_i}{}^*)$, where $(A_{N_i}{}^*)$'s are fuzzy neutrosophic supra σ-nowhere dense sets in (X_N*,T_N*). Any other fuzzy neutrosophic supra set in (X_N*,T_N*) is said to be a fuzzy neutrosophic supra σ-second category set in (X_N*,T_N*).

Definition 4.2:

Let ${A_N}^*$ be a fuzzy neutrosophic supra σ-first category set in a fuzzy neutrosophic supra topological space (X_N*,T_N*). Then $1 - {A_N}^*$ is called a fuzzy neutrosophic supra σ-residual set in (X_N*,T_N*).

Proposition 4.3:

If ${A_N}^*$ is a fuzzy neutrosophic supra σ-first category set in a fuzzy neutrosophic supra topological space (X_N*,T_N*), then there is a fuzzy neutrosophic supra F_σ-set ${C_N}^*$ in (X_N*,T_N*) such that ${A_N}^* \leq {C_N}^*$.

Proof.

Let ${A_N}^*$ be a fuzzy neutrosophic supra σ-first category set in (X_N*,T_N*). Then ${A_N}^* = \vee_{i=1}^{\infty}({A_{N_i}}^*)$, where $({A_{N_i}}^*)$'s are fuzzy neutrosophic supra σ-nowhere dense set in (X_N*,T_N*). Now $\left(1 - cl^*({A_{N_i}}^*)\right)$'s are fuzzy neutrosophic supra open sets in (X_N*,T_N*). Then, ${B_N}^* = \wedge_{i-1}^{\infty}\left(1 - cl^*({A_{N_i}}^*)\right)$ is a fuzzy neutrosophic supra G_δ-set in (X_N*,T_N*) and $1 - {B_N}^* = 1 - \left[\wedge_{i-1}^{\infty}\left(1 - cl^*({A_{N_i}}^*)\right)\right] = \vee_{i=1}^{\infty}\left(cl^*({A_{N_i}}^*)\right)$ but $\vee_{i=1}^{\infty}({A_{N_i}}^*) \leq \vee_{i=1}^{\infty}\left(cl^*({A_{N_i}}^*)\right)$, implies that ${A_N}^* \leq 1 - {B_N}^*$. Since ${B_N}^*$ is a fuzzy neutrosophic supra G_δ-set, $1 - {B_N}^*$ is a fuzzy neutrosophic supra F_σ-set in (X_N*,T_N*). Let ${C_N}^* \leq 1 - {B_N}^*$. Hence, if ${A_N}^*$ is a fuzzy neutrosophic supra σ-first category set in (X_N*,T_N*), then there is a fuzzy neutrosophic supra F_σ-set ${C_N}^*$ in (X_N*,T_N*) such that ${A_N}^* \leq {C_N}^*$.

Proposition 4.4:

If ${A_N}^*$ is a fuzzy neutrosophic supra σ-first category set in a fuzzy neutrosophic supra topological space (X_N*,T_N*), then there is a fuzzy neutrosophic supra G_δ-set ${B_N}^*$ in (X_N*,T_N*) such that $1 - {A_N}^* \geq {B_N}^*$.

Proof.

Let ${A_N}^*$ be a fuzzy neutrosophic supra σ-first category set in (X_N*,T_N*). Then by Proposition 4.3, there is a fuzzy neutrosophic supra F_σ-set ${C_N}^*$ in (X_N*,T_N*) such that ${A_N}^* \leq {C_N}^*$. Then $1 - {C_N}^* \leq 1 - {A_N}^*$. Since ${C_N}^*$ is a fuzzy neutrosophic supra F_σ-set $1 - {C_N}^*$ is a fuzzy neutrosophic supra G_δ-set in (X_N*,T_N*). Let ${B_N}^* \leq 1 - {C_N}^*$. Hence, if ${A_N}^*$ is a fuzzy neutrosophic supra σ-first category set in (X_N*,T_N*), then there is a fuzzy neutrosophic supra G_δ-set ${B_N}^*$ in (X_N*,T_N*) such that $1 - {A_N}^* \geq {B_N}^*$.

Proposition 4.5:

If ${A_N}^*$ is a fuzzy neutrosophic supra σ-nowhere dense set in a fuzzy neutrosophic supra topological space (X_N*,T_N*), then ${A_N}^*$ is a fuzzy neutrosophic supra first category set in (X_N*,T_N*).

Proof.

Let $A_N{}^*$ is a fuzzy neutrosophic supra σ-nowhere dense set in $(X_N{}^*,T_N{}^*)$, then $A_N{}^*$ is a fuzzy neutrosophic supra F_σ-set in $(X_N{}^*,T_N{}^*)$ such that $int^*(A_N{}^*) = 0$. Since $A_N{}^*$ is a fuzzy neutrosophic supra F_σ-set, $A_N{}^* = \vee_{i=1}^{\infty}(A_{N_i}{}^*)$, where $(A_{N_i}{}^*)$'s are fuzzy neutrosophic supra closed sets in $(X_N{}^*,T_N{}^*)$. Now $int^*(A_N{}^*) = 0$ implies that $int^*(\vee_{i=1}^{\infty}(A_{N_i}{}^*)) = 0$. But $\vee_{i=1}^{\infty} int^*(A_{N_i}{}^*) \leq int^*(\vee_{i=1}^{\infty}(A_{N_i}{}^*))$. This implies that $\vee_{i=1}^{\infty} int^*(A_{N_i}{}^*) \leq 0$. That is $\vee_{i=1}^{\infty} int^*(A_{N_i}{}^*) = 0$ and hence $int^*(A_{N_i}{}^*) = 0$ for each i. Since $(A_{N_i}{}^*)$'s are fuzzy neutrosophic supra closed sets in $(X_N{}^*,T_N{}^*)$, $cl^*(A_{N_i}{}^*) = A_{N_i}{}^*$. Then $int^*cl^*(A_{N_i}{}^*) = int^*(A_{N_i}{}^*) = 0$. Hence $(A_{N_i}{}^*)$'s are fuzzy neutrosophic supra nowhere dense set in $(X_N{}^*,T_N{}^*)$. This implies that $A_N{}^* = \vee_{i=1}^{\infty}(A_{N_i}{}^*)$, where $(A_{N_i}{}^*)$'s are fuzzy neutrosophic supra nowhere dense set in $(X_N{}^*,T_N{}^*)$. Therefore $A_N{}^*$ is a fuzzy neutrosophic supra first category set in $(X_N{}^*,T_N{}^*)$.

Proposition 4.6:

If $A_N{}^*$ is a fuzzy neutrosophic supra σ-nowhere dense set in a fuzzy neutrosophic supra topological space $(X_N{}^*,T_N{}^*)$, then $1 - A_N{}^*$ is a fuzzy neutrosophic supra residual set in $(X_N{}^*,T_N{}^*)$.

Proof.

Let $A_N{}^*$ is a fuzzy neutrosophic supra σ-nowhere dense set in $(X_N{}^*,T_N{}^*)$, then by Proposition 4.5, $A_N{}^*$ is a fuzzy neutrosophic supra first category set in $(X_N{}^*,T_N{}^*)$. Therefore $1 - A_N{}^*$ is a fuzzy neutrosophic supra residual set in $(X_N{}^*,T_N{}^*)$.

Proposition 4.7:

If $A_N{}^*$ is a fuzzy neutrosophic supra σ-nowhere dense set in a fuzzy neutrosophic supra topological space $(X_N{}^*,T_N{}^*)$, then there is a fuzzy neutrosophic supra F_σ-set $C_N{}^*$ in $(X_N{}^*,T_N{}^*)$ such that $A_N{}^* \leq C_N{}^*$.

Proof.

Let $A_N{}^*$ be a fuzzy neutrosophic supra σ-nowhere dense set in $(X_N{}^*,T_N{}^*)$, then by Proposition 4.5, $A_N{}^*$ is a fuzzy neutrosophic supra first category set in $(X_N{}^*,T_N{}^*)$. Then there is a fuzzy neutrosophic supra F_σ-set $C_N{}^*$ in $(X_N{}^*,T_N{}^*)$ such that $A_N{}^* \leq C_N{}^*$.

5. Fuzzy Neutrosophic Supra Dense G_δ-Spaces

Definition 5.1:

A fuzzy neutrosophic supra topological space $(X_N{}^*,T_N{}^*)$ is called a fuzzy neutrosophic supra Dense G_δ-space if each fuzzy neutrosophic supra dense set in $(X_N{}^*,T_N{}^*)$ is a fuzzy neutrosophic G_δ-set in $(X_N{}^*,T_N{}^*)$.

Example 5.2:

Let $X_N{}^* = \{a, b, c\}$. The fuzzy neutrosophic supra sets $A_N{}^*$, $B_N{}^*$ and $C_N{}^*$ are defined on $X_N{}^*$ as follows:

$$A_N{}^* = \{\langle a, 0.8, 0.6, 0.7\rangle,\ \langle b, 0.7, 0.6, 0.8\rangle,\ \langle c, 0.6, 0.5, 0.5\rangle\}$$

$$B_N{}^* = \{\langle a, 0.6, 0.7, 0.6\rangle, \langle b, 0.6, 0.5, 0.7\rangle, \langle c, 0.5, 0.6, 0.7\rangle\}$$

$$C_N{}^* = \{\langle a, 0.6, 0.8, 0.6\rangle, \langle b, 0.8, 0.8, 0.8\rangle, \langle c, 0.7, 0.7, 0.6\rangle\}$$

Now, $T_N{}^* = \{0, A_N{}^*, B_N{}^*, C_N{}^*, A_N{}^* \vee B_N{}^*, B_N{}^* \vee C_N{}^*, A_N{}^* \vee C_N{}^*, A_N{}^* \wedge B_N{}^*, B_N{}^* \wedge C_N{}^*, A_N{}^* \wedge C_N{}^*, A_N{}^* \vee (B_N{}^* \wedge C_N{}^*), B_N{}^* \vee (C_N{}^* \wedge A_N{}^*), A_N{}^* \vee (B_N{}^* \wedge C_N{}^*), A_N{}^* \vee B_N{}^* \vee C_N{}^*, 1\}$, is a fuzzy neutrosophic supra topology on $X_N{}^*$, On computation, we see that the fuzzy neutrosophic supra dense set in (X_N*,T_N*) is the fuzzy neutrosophic supra G_δ-set in (X_N*,T_N*). Now $E_N{}^* = [A_N{}^* \vee B_N{}^*] \wedge [B_N{}^* \vee C_N{}^*] \wedge [C_N{}^* \vee A_N{}^*] \wedge [A_N{}^* \vee (B_N{}^* \wedge C_N{}^*)] \wedge [B_N{}^* \vee (C_N{}^* \wedge A_N{}^*)] \wedge [A_N{}^* \vee (B_N{}^* \wedge C_N{}^*)]$, and hence the fuzzy neutrosophic supra dense set $E_N{}^*$ in (X_N*,T_N*) is a fuzzy neutrosophic supra dense G_δ- set in (X_N*,T_N*). Hence (X_N*,T_N*) is a fuzzy neutrosophic supra Dense G_δ-space.

Proposition 5.3:

If $A_N{}^*$ is a fuzzy neutrosophic supra dense set in a fuzzy neutrosophic supra Dense G_δ-space in (X_N*,T_N*), then $A_N{}^* = \wedge_{i=1}^{\infty}(B_{N_i}{}^*)$, where $(B_{N_i}{}^*)$'s are fuzzy neutrosophic supra dense and fuzzy neutrosophic supra open sets in (X_N*,T_N*).

Proof.

Let $A_N{}^*$ be a fuzzy neutrosophic supra dense set in (X_N*,T_N*). Then $cl(A_N{}^*) = 1$. Since (X_N*,T_N*) is a fuzzy neutrosophic supra G_δ-space. $A_N{}^*$ is fuzzy neutrosophic supra G_δ-set in (X_N*,T_N*). Then $A_N{}^* = \wedge_{i=1}^{\infty}(A_{N_i}{}^*)$, where $(A_{N_i}{}^*)$'s are fuzzy neutrosophic supra open sets in (X_N*,T_N*). Now $cl(A_N{}^*) = cl[\wedge_{i=1}^{\infty}(A_{N_i}{}^*)]$ in (X_N*,T_N*). But $cl[\wedge_{i=1}^{\infty}(A_{N_i}{}^*)] = [\wedge_{i=1}^{\infty} cl(A_{N_i}{}^*)]$, implies that $1 = [\wedge_{i=1}^{\infty} cl(A_{N_i}{}^*)]$. That is,$\wedge_{i=1}^{\infty} cl(A_{N_i}{}^*) = 1$ and then $cl(A_{N_i}{}^*) = 1$, in (X_N*,T_N*). Hence, $A_N{}^* = \wedge_{i=1}^{\infty}(A_{N_i}{}^*)$, where $(A_{N_i}{}^*)$'s are fuzzy neutrosophic supra dense and fuzzy neutrosophic supra open sets in (X_N*,T_N*).

Proposition 5.4:

If $A_N{}^*$ is a fuzzy neutrosophic supra dense sets in a fuzzy neutrosophic supra Dense G_δ-space (X_N*,T_N*), then $A_N{}^*$ is a fuzzy neutrosophic supra residual set in (X_N*,T_N*).

Proof.

Let $A_N{}^*$ is a fuzzy neutrosophic supra dense sets in (X_N*,T_N*). Since (X_N*,T_N*) is a fuzzy neutrosophic supra Dense G_δ-space Proposition 5.3, then $A_N{}^* = \wedge_{i=1}^{\infty}(A_{N_i}{}^*)$, where $(A_{N_i}{}^*)$ are fuzzy neutrosophic dense and fuzzy neutrosophic supra open sets in (X_N*,T_N*). Then $1 - A_N{}^* = 1 - \wedge_{i=1}^{\infty}(A_{N_i}{}^*) = \vee_{i-1}^{\infty}(1 - A_{N_i}{}^*)$. Now $int\, cl(1 - A_{N_i}{}^*) = 1 - cl\, int(A_{N_i}{}^*) = 1 - cl(A_{N_i}{}^*) = 1 - 1 = 0$ and thus $(1 - A_{N_i}{}^*)$'s fuzzy neutrosophic supra nowhere dense sets in (X_N*,T_N*). Hence $1 - A_N{}^* =$

$\vee_{i-1}^{\infty}(1 - A_{N_i}^{*})$, where$(1 - A_{N_i}^{*})$'s are fuzzy neutrosophic supra nowhere dense sets in (X_N*,T_N*), implies that $1 - A_N^{*}$ is a fuzzy neutrosophic supra first category set in (X_N*,T_N*). Thus A_N^{*} is a fuzzy neutrosophic supra residual set in (X_N*,T_N*).

Proposition 5.5:

If A_N^{*} is a fuzzy neutrosophic supra nowhere dense sets in a fuzzy neutrosophic supra Dense G_δ-space (X_N*,T_N*), then A_N^{*} is a fuzzy neutrosophic supra F_σ-set in (X_N*,T_N*).

Proof.

Let A_N^{*} is a fuzzy neutrosophic supra nowhere dense sets in (X_N*,T_N*). Then, $int\ cl(A_N^{*}) = 0$, in (X_N*,T_N*). But $int(A_N^{*}) \leq\ int\ cl(A_N^{*})$ implies that $int^{*}(A_N^{*}) = 0$ in (X_N*,T_N*). Now $cl(1 - A_N^{*}) = 1 - int(A_N^{*}) = 1 - 0 = 1$ and hence $1 - A_N^{*}$ is a fuzzy neutrosophic dense set in (X_N*,T_N*). Since (X_N*,T_N*) is a fuzzy neutrosophic supra Dense G_δ-space, the fuzzy neutrosophic dense set $1 - A_N^{*}$ is a fuzzy neutrosophic supra G_δ-set in (X_N*,T_N*). Hence A_N^{*} is a fuzzy neutrosophic supra F_σ-set in (X_N*,T_N*).

Proposition 5.6:

If A_N^{*} is a fuzzy neutrosophic supra nowhere dense sets in a fuzzy neutrosophic supra Dense G_δ-space (X_N*,T_N*), then A_N^{*} is a fuzzy neutrosophic supra σ –nowhere dense set in (X_N*,T_N*).

Proof.

Let (X_N*,T_N*) be a fuzzy neutrosophic supra Dense G_δ-space and A_N^{*} be a fuzzy neutrosophic supra nowhere dense set in (X_N*,T_N*). The, by Proposition 5.5, A_N^{*} is a fuzzy neutrosophic supra F_σ-set in (X_N*,T_N*). Now, $int(A_N^{*}) \leq int\ cl(A_N^{*})$ and $int\ cl(A_N^{*}) = 0$, implies that $int\ (A_N^{*}) = 0$ in (X_N*,T_N*). Thus, Hence A_N^{*} is a fuzzy neutrosophic supra F_σ-set such that $int\ (A_N^{*}) = 0$ in (X_N*,T_N*). Therefore A_N^{*} is a fuzzy neutrosophic supra σ –nowhere dense set in (X_N*,T_N*).

Proposition 5.7:

If A_N^{*} is a fuzzy neutrosophic supra σ-nowhere dense set in a fuzzy neutrosophic supra Dense G_δ-space (X_N*,T_N*), then there exists a fuzzy neutrosophic supra F_σ-set E_N^{*} in (X_N*,T_N*) such that $A_N^{*} \leq E_N^{*}$.

Proof.

Let A_N^{*} is a fuzzy neutrosophic supra σ-nowhere dense set in (X_N*,T_N*). Then by Proposition 4.3, $A_N^{*} \leq 1 - C_N^{*}$. Let $E_N^{*} = 1 - C_N^{*}$. Then E_N^{*} is a fuzzy neutrosophic supra F_σ-set in (X_N*,T_N*). Thus, if A_N^{*} is a fuzzy neutrosophic supra σ-nowhere dense set in (X_N*,T_N*), then there exists a fuzzy neutrosophic supra F_σ-set E_N^{*} in (X_N*,T_N*) such that $A_N^{*} \leq E_N^{*}$.

Proposition 5.8:

If $int^{*}(\vee_{i=1}^{\infty}(A_{N_i}^{*})) = 0$, where $(A_{N_i}^{*})$,s are fuzzy neutrosophic supra nowhere dense set in a fuzzy neutrosophic supra Dense G_δ-space in (X_N*,T_N*), then (X_N*,T_N*) is a fuzzy neutrosophic supra σ-nowhere dense set.

Proof.

Suppose that $int^*\left(\vee_{i=1}^{\infty}\left({A_{N_i}}^*\right)\right) = 0$, where $\left({A_{N_i}}^*\right)$,s are fuzzy neutrosophic supra nowhere dense set in a fuzzy neutrosophic supra Dense G_δ-space (X_N*,T_N*). Since $\left({A_{N_i}}^*\right)$'s are fuzzy neutrosophic supra nowhere dense set in the fuzzy neutrosophic supra Dense G_δ-space (X_N*,T_N*), by Proposition 5.6, $\left({A_{N_i}}^*\right)$'s are fuzzy neutrosophic supra nowhere dense set in (X_N*,T_N*).

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