INNOVATIONS

Reliability of time dependent stress- strength system for deterministic cycles

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Abstract:
Sometimes a system can work only when the stress on it lies between certain values. There is uncertainty about the stress and strength random variables with respect to time and cycles. These uncertainties may be classified as deterministic, random fixed and random independent In this paper, reliability of time dependent stress strength system has been derived when stress and strength follow generalized exponential distribution, generalized weibull distribution and pareto type-I distribution for different cases i.e. i) deterministic stress and random fixed strength, ii) random – fixed stress and deterministic strength, iii) deterministic stress and random – independent strength. Numerical computations have been obtained for various uncertainties of stress and strength models.

Key words: 1.Reliability 2. stress- strength system 3. generalized exponential distribution 4.generalized weibull distribution 5. pareto type- I distribution.

Introduction:
Reliability of the stress- strength model is defined as P(X>Y) where X denotes strength random variable and Y denotes stress random variable. Chumchum Doloi et al discussed the cascade system with P(X<Y<Z) when stress and strength follow different distributions. M.N. Gopalan and P.Venkateswarlu discussed the reliability analysis of time dependent cascade system with deterministic cycle times.

Statistical Model:
If $X_i, Y_i\ and\ Z_i$ follow generalized exponential distribution then the probability density function of $X_i, Y_i\ and\ Z_i$ are $f(x_i), g(y_i) and\ h(z_i)$ respectively then

$$f(x_i, \alpha, \lambda) = \alpha\left(1 - e^{-\lambda x_i}\right)^{\alpha-1} e^{-x_i\lambda}, x_i > 0$$
$$g(y_i, \alpha, \mu) = \alpha(1 - e^{-\mu y_i})^{\alpha-1} e^{-y_i\mu}, y_i > 0$$
$$h(z_i, \alpha, \gamma) = \alpha(1 - e^{-\gamma z_i})^{\alpha-1} e^{-z_i\gamma}, z_i > 0$$

If $X_i, Y_i\ and\ Z_i$ follow generalized weibull distribution then the probability density function of $X_i, Y_i\ and\ Z_i$ are $f(x_i), g(y_i) and\ h(z_i)$ respectively then

$$f(x_i, \alpha, \beta, \lambda) = \left(\alpha + \beta\lambda x^{\lambda-1}\right) e^{-\alpha x_i - \beta x_i{}^{\lambda}}, x_i > 0$$
$$g(y_i, \alpha, \beta, \mu) = (\alpha + \beta\mu x^{\mu-1}) e^{-\alpha x_i - \beta x_i{}^{\mu}}, y_i > 0$$
$$h(z_i, \alpha, \beta, \gamma) = (\alpha + \beta\gamma x^{\gamma-1}) e^{-\alpha x_i - \beta x_i{}^{\gamma}}, z_i > 0$$

If X_i, Y_i *and* Z_i follow pareto type-I distribution then the probability density function of X_i, Y_i *and* Z_i are $f(x_i), g(y_i)$*and* $h(z_i)$ respectively then

$$f(x_i, \alpha, \lambda) = \frac{\alpha\lambda^\alpha}{x^{\alpha+1}}, x_i > 0$$

$$g(y_i, \alpha, \mu) = \frac{\alpha\mu^\alpha}{y^{\alpha+1}}, y_i > 0$$

$$h(z_i, \alpha, \gamma) = \frac{\alpha\gamma^\alpha}{z^{\alpha+1}}, z_i > 0$$

Case 1: Deterministic stress and random - fixed strength

Let the stress y_0 , a constant and the strength on the i[th] cycle be X_i *and* Z_i given by

$$X_i = X_0 + a_i, Z_i = Z_0 - b_i, i = 1,2,3, ..$$

Then

$$R(i) = P[X_0 + a_i < Z_0 - b_i]$$

Where a_i *and* b_i non negative constants

Therefore Reliability of the i[th] cycle

$$R(i) = P[Y_0 > X_0 + a_i] - P[Y_0 > X_0 + a_i, Y_0 > Z_0 - b_i]$$

$$= P[Y_0 - a_i > X_0] - P[Y_0 - a_i > X_0, Y_0 + b_i > Z_0]$$

$$= \int_{x_0=0}^{y_0-a_i} f(x_0)dx_0 - \left(\int_{x_0=0}^{y_0-a_i} f(x_0)dx_0 \int_{z_0=0}^{y_0+b_i} h(z_0)dz_0\right)$$

$$= \int_{x_0=0}^{y_0-a_i} f(x_0)dx_0 \left[1 - \int_{z_0=0}^{y_0+b_i} h(z_0)dz_0\right]$$

If X_i, Y_i *and* Z_i follow generalized exponential distribution, then

$$R(i) = \frac{\left(1 - e^{-\lambda(y_0-a_i)}\right)^\alpha \left[1 - \left(1 - e^{-\gamma(y_0+b_i)}\right)^\alpha\right]}{\lambda\gamma}$$

If X_i, Y_i *and* Z_i follow generalized weibull distribution, then

$$R(i) = \left[1 - e^{-(\lambda(y_0-a_i))^\beta}\right]^\alpha \left[1 - \left(1 - e^{-(\lambda(y_0+b_i))}\right)^\beta\right]^\alpha$$

If X_i, Y_i *and* Z_i follow pareto type –I distribution, then

$$R(i) = \frac{-\lambda^\alpha}{(y_0 - a_i)^\alpha}\left[1 + \frac{\gamma^\alpha}{(y_0 + b_i)^\alpha}\right]$$

Case 2: Random fixed stress and deterministic strength:

Let the random fixed stress of the i[th] is $Y_0 - a_i$ and deterministic strength be X_0 *and* Z_0 then

$$R(i) = P[X_0 < Y_0 - a_i < Z_0]$$

$$= P[X_0 + a_i < Y_0 < Z_0 + a_i]$$

$$= \int_{y_0=x_0+a_i}^{z_0+a_i} g(y_0)dy_0$$

If X_i, Y_i *and* Z_i follow generalized exponential distribution, then

$$R(i) = \frac{\left[e^{-\mu(x_0+a_i)} - e^{-\mu(z_0+a_i)}\right]}{\mu}$$

If X_i, Y_i *and* Z_i follow generalized weibull distribution, then

$$R(i) = \left[e^{-(\mu(x_0+a_i))^\beta} - e^{-(\mu(z_0+a_i))^\beta}\right]$$

If X_i, Y_i *and* Z_i follow pareto type –I distribution, then

$$R(i) = \mu^\alpha[(x_0 + a_i)^{-\alpha} - (z_0 + a_i)^{-\alpha}]$$

Case 3: Deterministic stress and random independent strength:

Let the deterministic stress be Y_0 *and* X_i *and* Z_i are random independent strengths then

$$P[E_i] = P[X_i < Y_0 > Z_i]$$
$$= P[Y_0 > X_i] - P[Y_0 > X_i, Y_0 > Z_i\,]$$
$$= \int_{x_i=0}^{y_0} f(x_i)\,dx_i - \left[\int_{x_i=0}^{y_0} f(x_i)\,dx_i * \int_{z_i=0}^{y_0} h(z_i)dz_i\right]$$
$$= \int_{x_i=0}^{y_0} f(x_i)\,dx_i\left[1 - \int_{z_i=0}^{y_0} h(z_i)dz_i\right]$$

If X_i, Y_i *and* Z_i follow generalized exponential distribution, then

$$P(E_i) = \frac{1}{\lambda}\left(1 - e^{-y_0\lambda}\right)^{\alpha}\left(1 - \frac{(1 - e^{-y_0\gamma})^{\alpha}}{\gamma}\right)$$

Therefore

$$R(i) = P[E_1, E_2, \dots E_n] = \prod_{i=1}^{n} P(E_i)$$
$$= \frac{1}{\lambda}\left(1 - e^{-y_0\lambda}\right)^{i\alpha}\left(1 - \frac{(1 - e^{-y_0\gamma})^{\alpha}}{\gamma}\right)^{i}$$

If X_i, Y_i *and* Z_i follow generalized weibull distribution, then

$$P(E_i) = \left(1 - e^{-(\lambda y_0)^{\beta}}\right)^{\alpha}\left(1 - \left[1 - e^{-(\gamma y_0)^{\beta}}\right]^{\alpha}\right)$$
$$R(i) = P[E_1, E_2, \dots E_n] = \prod_{i=1}^{n} P(E_i)$$
$$= \left(1 - e^{-(\lambda y_0)^{\beta}}\right)^{\alpha i}\left(1 - \left[1 - e^{-(\gamma y_0)^{\beta}}\right]^{\alpha}\right)^{i}$$

If X_i, Y_i *and* Z_i follow pareto type –I distribution, then

$$P(E_i) = -\lambda^{\alpha}[{y_0}^{-\alpha} + \gamma^{\alpha}{y_0}^{-2\alpha}]$$
$$R(i) = P[E_1, E_2, \dots E_n] = \prod_{i=1}^{n} P(E_i)$$
$$= \lambda^{i\alpha}[{y_0}^{-\alpha} + \gamma^{\alpha}{y_0}^{-2\alpha}]^{i}$$

Numerical Analysis:

Case i) Deterministic stress and random fixed strength:

Generalized Exponential Distribution:

Table 1 $(y_0 = 0.5, a_i = 0.05, b_i = 0.2, \alpha = 1, \gamma = 1.5)$

λ	1	2	3	4	5	6	7	8	9	10
R	0.1902	0.3115	0.3888	0.4381	0.4696	0.4896	0.5024	0.5106	0.5158	0.5191

Table 2 $(y_0 = 0.5, a_i = 0.05, b_i = 0.2, \lambda = 2, \gamma = 1.5)$

α	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
R	0.2106	0.3114	0.3525	0.3615	0.3539	0.3383	0.3194	0.2996	0.2803	0.2622

Table 3 $(y_0 = 0.5, a_i = 0.05, b_i = 0.2, \lambda = 2, \alpha = 1)$

γ	1	2	3	4	5	6	7	8	9	10
R	0.4442	0.4412	0.3286	0.2176	0.1351	0.0805	0.0466	0.02647	0.0148	0.0082

Table 4 $(\gamma = 1.5, a_i = 0.05, b_i = 0.2, \lambda = 5, \alpha = 1)$

y_0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
R	0	0.2116	0.3491	0.4344	0.4828	0.5055	0.5107	0.5039	0.4892	0.4696

Table 5 $(\gamma = 1.5, y_0 = 0.5, b_i = 0.2, \lambda = 5, \alpha = 1)$

a_i	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
R	0.4796	0.4773	0.4748	0.4722	0.4696	0.4667	0.4638	0.4606	0.4573	0.4539

Generalized Weibull Distribution:

Table 1 $(y_0 = 0.5, a_i = 0.05, b_i = 0.2, \alpha = 0.1, \beta = 2)$

λ	1	2	3	4	5	6	7	8	9	10
R	0.8196	0.8238	0.7979	0.7614	0.7208	0.6793	0.6383	0.5987	0.5608	0.5247

Table 2 $(y_0 = 0.5, a_i = 0.05, b_i = 0.2, \lambda = 5, \beta = 2)$

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
R	0.7208	0.5195	0.3745	0.2699	0.1946	0.1402	0.1011	0.0729	0.0525	0.0379

Table 3 $(y_0 = 0.5, a_i = 0.05, b_i = 0.2, \lambda = 2, \alpha = 0.1)$

β	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
R	0.7923	0.8256	0.8309	0.8238	0.8094	0.7906	0.7690	0.7459	0.7221	0.6979

Table 4 $(\beta = 2, a_i = 0.05, b_i = 0.2, \lambda = 2, \alpha = 0.1)$

y_0	1	2	3	4	5	6	7	8	9	10
R	0.8243	0.6898	0.5651	0.4627	0.3788	0.3101	0.2539	0.2079	0.1702	0.1394

Pareto Type-I distribution:

Table 1 $(y_0 = 0.1, a_i = 0.5, \alpha = 2, \gamma = 1)$

λ	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
R	0.0076	0.03029	0.06813	0.1211	0.01892	0.2725	0.3709	0.4844	0.6131	0.7569

Table 2 $(y_0 = 0.1, a_i = 0.5, \lambda = 0.01, \alpha = 2)$

γ	1	2	3	4	5	6	7	8	9	10
R	0.0076	0.0284	0.0631	0.1117	0.1742	0.2506	0.3409	0.4451	0.5631	0.6951

Table 3 $(\gamma = 5, a_i = 2, \lambda = 0.01, \alpha = 2)$

y_0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
R	0.0077	0.0049	0.0035	0.0028	0.0023	0.0021	0.0019	0.0018	0.0017	0.0016

Table 4 $(\gamma = 2, a_i = 1, \lambda = 0.1, y_0 = 0.5)$

α	1	2	3	4	5	6	7	8	9	10
R	0.7714	0.3665	0.1946	0.1082	0.0612	0.0349	0.0199	0.0114	0.0065	0.0037

Case 2: Random fixed stress and deterministic strength:

Generalized Exponential Distribution:

Table 1 $(x_0 = 0.1, a_i = 0.1, z_0 = 5)$

μ	1	2	3	4	5	6	7	8	9	10
R	0.8126	0.3351	0.1829	0.1123	0.0736	0.0502	0.0352	0.0252	0.0184	0.0135

Table 2 $(\mu = 1, a_i = 0.1, z_0 = 5)$

x_0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
R	0.8126	0.7347	0.6642	0.6004	0.5427	0.4905	0.4432	0.4004	0.3618	0.3268

Table 3 $(\mu = 1, a_i = 0.1, x_0 = 0.1)$

z_0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
R	0.2699	0.4857	0.6168	0.6963	0.7446	0.7736	0.7914	0.8022	0.8087	0.8126

Table 4 $(\mu = 1, z_0 = 5, x_0 = 0.1)$

a_i	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
R	0.8126	0.7353	0.6653	0.6020	0.5447	0.4929	0.4459	0.4035	0.3654	0.3304

Generalized Weibull Distribution:

Table 1 $(x_0 = 0.5, z_0 = 5, a_i = 0.5, \beta = 1)$

μ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
R	0.3279	0.4856	0.5488	0.5595	0.5426	0.5119	0.4753	0.4371	0.3995	0.3638

Table 2 $(\mu = 0.1, z_0 = 5, a_i = 0.5, \beta = 1)$

x_0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
R	0.3648	0.3554	0.3462	0.3369	0.3279	0.3188	0.3099	0.3011	0.2924	0.2836

Table 3 $(\mu = 0.1, x_0 = 0.5, a_i = 0.5, \beta = 1)$

z_0	1	2	3	4	5	6	7	8	9	10
R	0.0441	0.1260	0.2001	0.2672	0.3279	0.3828	0.4325	0.4774	0.5181	0.5549

Table 4 $(\mu = 0.1, x_0 = 0.5, z_0 = 0.5, \beta = 1)$

a_i	1	2	3	4	5	6	7	8	9	10
R	0.3119	0.2822	0.2553	0.2311	0.2091	0.1892	0.1712	0.1549	0.1401	0.1268

Pareto Type-I distribution:

Table 1 $(x_0 = 0.1, z_0 = 2, a_i = 0.1, \alpha = 0.5)$

μ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
R	0.2488	0.3519	0.4310	0.4977	0.5564	0.6096	0.6584	0.7038	0.7465	0.7869

Table 2 $(\mu = 0.1, z_0 = 1.5, a_i = 0.01, \alpha = 0.6)$

x_0	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55
R	0.9439	0.6557	0.4836	0.3668	0.2812	0.2153	0.1625	0.1191	0.0827	0.0516

Table 3 $(\mu = 0.2, x_0 = 0.1, a_i = 0.1, \alpha = 0.5)$

z_0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
R	0.8	0.7551	0.7172	0.6838	0.6536	0.6258	0.6	0.5757	0.5528	0.5309

Table 4 ($\mu = 0.2, x_0 = 0.1, z_0 = 0.5, \alpha = 0.5$)

a_i	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55
R	0.6536	0.5338	0.4423	0.3686	0.3071	0.2544	0.2082	0.1671	0.1301	0.0964

Table 5 ($\mu = 0.2, x_0 = 0.1, z_0 = 0.5, a_i = 0.1$)

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
R	0.1911	0.3456	0.4706	0.5718	0.6536	0.7198	0.7732	0.8166	0.8517	0.88

Case 3: Deterministic stress and random independent strength:

Generalized Exponential Distribution:

Table 1 ($y_0 = 0.5, i = 1, \alpha = 0.6, \gamma = 1.5$)

λ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
R	0.891	0.6653	0.5574	0.4896	0.4414	0.4045	0.4046	0.3750	0.3297	0.3118

Table 2($\lambda = 0.2, i = 1, \alpha = 0.6, \gamma = 1.5$)

y_0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
R	0.3782	0.5037	0.5788	0.6290	0.6653	0.6930	0.7153	0.7342	0.7509	0.7663

Table 3($\lambda = 0.2, y_0 = 2, \alpha = 0.2, \gamma = 2$)

i	1	2	3	4	5	6	7	8	9	10
R	0.9157	0.4193	0.1919	0.0879	0.0402	0.0184	0.0084	0.0039	0.0017	0.0008

Table 4($\lambda = 0.2, y_0 = 0.5, i = 1, \gamma = 1.2$)

α	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
R	0.9036	0.7686	0.5889	0.4258	0.2969	0.2019	0.1349	0.0889	0.0580	0.0375

Table 5 ($\lambda = 0.2, y_0 = 0.5, i = 1, \alpha = 1.2$)

γ	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
R	0.6303	0.6790	0.7221	0.7607	0.7954	0.8270	0.8558	0.8823	0.9066	0.9292

Generalized Weibull Distribution:

Table 1 ($y_0 = 0.5, i = 1, \alpha = 0.5, \gamma = 1.5, \beta = 3$)

λ	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8
R	0.3389	0.4358	0.5328	0.6259	0.7114	.7861	0.8482	0.8967	0.9322	0.9565

Table 2 ($\lambda = 2, i = 1, \alpha = 0.5, \gamma = 0.1, \beta = 3$)

y_0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
R	0.0892	0.2482	0.4384	0.6279	0.7861	0.8935	0.9493	0.9692	0.9715	0.9682

Table 3 ($\lambda = 2, y_0 = 0.5, \alpha = 0.5, \gamma = 0.1, \beta = 3$)

i	1	2	3	4	5	6	7	8	9	10
R	0.7862	0.6180	0.4859	0.3820	0.3003	0.2361	0.1856	0.1459	0.1147	0.0902

Table 4 ($\lambda = 2, y_0 = 0.5, i = 1, \gamma = 0.5, \beta = 3$)

α	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5
R	0.6223	0.5016	0.3995	0.3177	0.2526	0.2008	0.1597	0.1269	0.1009	0.0802

Table 5 ($\boldsymbol{\lambda = 2, y_0 = 0.5, i = 1, \beta = 3, \alpha = 0.5}$)

γ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
R	0.7862	0.7699	0.7489	0.7241	0.6960	0.6653	0.6322	0.5971	0.5604	0.5225

Pareto Type-I distribution:

Table 1 ($\boldsymbol{\lambda = 0.2, y_0 = 1.5, i = 1, \alpha = 0.5}$)

γ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
R	0.4594	0.4984	0.5284	0.5537	0.5759	0.5960	0.6146	0.6318	0.6480	0.6633

Table 2 ($\boldsymbol{\lambda = 0.2, y_0 = 1.5, i = 1, \gamma = 0.1}$)

α	0.25	0.5	0.75	1	0.25	1.5	1.75	2	2.25	2.5
R	0.9113	0.4594	0.2496	0.1422	0.0833	0.0495	0.0296	0.0178	0.0108	0.0065

Table 3 ($\boldsymbol{\lambda = 0.2, y_0 = 1.5, \gamma = 0.1, \alpha = 0.5}$)

i	1	2	3	4	5	6	7	8	9	10
R	0.4594	0.20555	0.0918	0.0411	0.0184	0.0082	0.0037	0.0016	0.0007	0.0003

Table 4 ($\boldsymbol{\lambda = 0.2, \gamma = 0.1, i = 1, \alpha = 0.5}$)

y_0	1	2	3	4	5	6	7	8	9	10
R	0.5886	0.3869	0.3053	0.2589	0.2283	0.2061	0.1892	0.1758	0.1648	0.1556

Table 5 ($\boldsymbol{\gamma = 0.1, y_0 = 1.5, i = 1, \alpha = 0.5}$)

λ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
R	0.3249	0.4594	0.5626	0.6497	0.7264	0.7958	0.8595	0.9188	0.9745	0.9954

Conclusion:

Reliability of time dependent stress strength system has been derived when stress and strength follow generalized exponential distribution, generalized weibull distribution and pareto type-I distribution for different cases i.e. i) deterministic stress and random fixed strength, ii) random – fixed stress and deterministic strength, iii) deterministic stress and random – independent strength. Numerical computations have been obtained for various uncertainties of stress and strength models.

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